

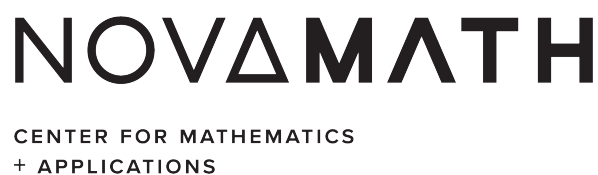


#optimization²⁰²⁶
JULY 20 – 22 · ISEG, Lisbon, Portugal



Book of Abstracts





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Welcome to Optimization 2026!

On behalf of the Organizing Committee and the Program Committee, it is our great pleasure to welcome you to Optimization 2026 at ISEG – Lisbon School of Economics and Management, Universidade de Lisboa. This is the 11th edition of a series of international conferences in optimization organized in Portugal under the auspices of APDIO, the Portuguese Operations Research Society. The conference is also sponsored by ISEG, ISEG-Research, CEMAPRE, Universidade de Lisboa, Nova-Math, FCT, LTPlabs, Hexaly, and SISCOG.

We are very pleased to announce that the conference will have 15 organized sessions, 21 contributed sessions, and six plenary sessions, with a total of 132 oral presentations and participants from 28 different countries.

We organized a social program that includes a Welcome Reception (July 20), a Historical Walking Tour followed by the Conference Dinner (July 21) in a restaurant located near the Tejo River, where you can enjoy a pleasant view.

We would like to extend our special thanks to the six plenary speakers for accepting our invitation and honoring us with their presence. We also thank all the researchers who submitted presentations for their interest and active participation. Our gratitude also goes to our conference sponsors. Last but not least, we warmly thank the members of the Organizing Committee, whose hard work, dedication, and patience with which they contributed to the success of this conference.

We wish all the participants an excellent conference!
Enjoy!

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	July 20 Monday	July 21 Tuesday	July 22 Wednesday
8.00-8:30	Registration		
8:30-9:00	Opening session		
9.00-10.00	Plenary Auditório CGD	Plenary Auditório CGD	Plenary Auditório CGD
10:00-10:30	Coffee Break	Coffee Break	Coffee Break
10:30-12:10	Parallel Sessions MA	Parallel Sessions TA	Parallel Sessions WA
12:10-14:00	Lunch Salão Nobre	Lunch Salão Nobre	Lunch Salão Nobre
14:00-15:00	Plenary Auditório CGD	Plenary Auditório CGD	Plenary Auditório CGD
15:00-15:10			Closing session
15:10-16:25	Parallel Sessions MB	Parallel Sessions TB	
16:25-17:00	Coffee Break	Coffee Break	
17:00-18:15	Parallel Sessions MC	Semi-Plenary & Parallel Sessions TC	
18:15-18:40			
18:40-19:00			
19:00-20:00	Welcome Reception Salão Nobre ISEG	18:40 Lisbon Walking Tour	Detailed Program: 
20:00-21:30		20:00 Conference Dinner - Monte Mar - Restaurant	
...			

Dick den Hertog

University of Amsterdam, The Netherlands

Title: Optimization for a Better World**Chair:** Cristina Requejo**Room:** CGD (Floor 2)**Abstract**

This keynote explores the transformative role of optimization in helping non-governmental organizations (NGOs) and non-profit organizations (NPOs) accelerate progress toward the United Nations Sustainable Development Goals (SDGs). Through the work of the Analytics for a Better World Institute, cutting-edge optimization methods are developed and applied to address some of society's most pressing humanitarian and environmental challenges. Drawing on real-world collaborations with NGOs and NPOs, the keynote will demonstrate how optimization can substantially enhance social impact. One featured example is a collaboration with The Ocean Cleanup, where advanced optimization models are used to determine how cleanup vessels should be deployed and routed to maximize the removal of plastic from the world's oceans, thereby increasing the effectiveness of large-scale efforts to combat marine pollution. The keynote will also present a range of projects that use geospatial optimization to improve access to healthcare and other essential services. These include optimizing the locations of primary healthcare facilities in Timor-Leste, determining the optimal placement of stroke centers in Vietnam, planning COVID-19 testing centers in Nepal, and identifying suitable locations for water wells in Sudan. These initiatives have been conducted in partnership with organizations such as the World Bank, the World Health Organization, Amref Health Africa, and the American Red Cross. Beyond showcasing the societal impact of these applications, the keynote will discuss the novel methodological and computational challenges they have generated. Many of these challenges remain open research questions, creating exciting opportunities for future advances in optimization and analytics for social good.

MA1: Routing I**Chair:** Leonor S. Pinto**Room:** Amplit. 1 (Floor 4)

Evaluating Hybrid Home Delivery and Self-Collection Strategies in Two-Echelon Urban Logistics

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Abstract

This paper investigates alternative last-mile delivery strategies aimed at reducing the environmental impact of urban logistics, with a particular focus on the management of failed deliveries. We consider a two-echelon distribution system in which parcels are transported from a central depot to pre-existing urban satellites and subsequently either delivered to customers' homes or made available for customer self-collection. An emission-aware Two-Echelon Location-Routing Problem is proposed, jointly optimizing satellite activation, vehicle routing, and the allocation of deliveries between home delivery and self-collection. The model includes (i) unattended deliveries, (ii) first-attempt attended deliveries, and (iii) attended deliveries that fail due to customer unavailability. The framework is used to evaluate alternative company-driven service strategies, comparing a baseline full home-delivery configuration with a hybrid strategy that enables satellite self-collection as a recovery option under both the minimization of total CO_2 emissions and total traveled distance. To efficiently solve large-scale realistic instances, an ALNS is implemented. Computational results on urban case-study instances show that emission-oriented hybrid strategies can significantly reduce CO_2 emissions while creating limited increases in traveled distance, highlighting the effectiveness of satellite self-collection as a lever for more sustainable last-mile delivery systems.

Keywords: *Last-mile delivery, Location-Routing problem, Environmental sustainability, Customer self-collection*

Acknowledgments: This work was supported by the European Union's Horizon Europe programme, Marie Skłodowska-Curie Actions (ERA Fellowship, grant agreement No. 101244242).

Safety-Aware Order Picking and Routing in Shared Human-Robot Warehouses

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Abstract

Warehouse automation increasingly involves shared spaces where humans and autonomous robots operate in parallel, raising concerns regarding efficiency and safety. This paper addresses the safety-aware order picking and routing problem in hybrid warehouses. Human actions are assumed autonomous, dynamic, and stochastic, whereas robot tasks—including scheduling and routing—are centrally planned.

The problem is modeled as a Route-based Markov Decision Process optimizing a bi-criteria objective that minimizes both total order completion time and human-robot collision probability. To overcome the model’s curse of dimensionality, we propose heuristic algorithms integrating Open Steiner Traveling Salesman and Identical Parallel Machine Scheduling Problems. We evaluate multiple routing strategies, from ignoring human presence to solving a Time-Dependent Shortest Path that leverages dynamic probabilistic data of future human presence to define immediate robot actions.

Through simulation, we analyze the trade-off between picking time and safety under different information settings, order consolidation levels, and warehouse layouts. Results show that incorporating probabilistic predictions reduces human-robot collisions by about 93%, with minimal impact on efficiency. Finally, we illustrate how tactical layout decisions offer safety benefits comparable to perfect information on human movement, highlighting how infrastructure optimization can compensate for information quality.

Keywords: *Human-robot collaboration; Safety-aware routing; Order picking*

Acknowledgments: The work of Andrea Mor and Carlotta Orsenigo has been developed within the HumanTech Project. The HumanTech Project is financed by the Italian Ministry of University and Research (MUR) for the 2023-2027 period as part of the ministerial initiative “Departments of Excellence” (L. 232/2016).

The Precedence Constrained Family Traveling Salesman Problem

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Abstract

We introduce the Precedence-Constrained Family Traveling Salesman Problem (PC-FTSP), which generalizes the FTSP by adding precedence relations between families. The PC-FTSP arises in warehouse order picking with scattered storage, where heavier items must be picked before lighter ones. We propose several formulations and compare them theoretically and empirically. We also derive valid inequalities that strengthen the linear programming (LP) relaxations. As some formulations are non-compact, we design a branch-and-cut algorithm to solve them. Results show the non-compact formulations outperform the compact ones, and the inequalities effectively strengthen the LP bounds on our instances.

Keywords: *Family Traveling Salesman Problem, Precedence Constraints, Projection Results, Branch-and-cut*

Acknowledgments: This work is financed by Portuguese funds through the FCT - Foundation for Science and Technology. I.P.. under projects UID/06522/2025 (ISEG Research) and UID/97/2025 (CEGIST).

MA2: Optimization and Wildfire Management

Chair: Isabel Martins

Room: Amphit. 2 (Floor 4)

Organizer: Filipe Alvelos & Isabel Martins

Predictive and Prescriptive Analytics toward Optimizing Wildfire Suppression

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Abstract

Intense wildfire seasons require critical prioritization decisions to suppress wildfires over a disperse area with limited resources. This paper develops a predictive and prescriptive approach to jointly optimize crew assignments and wildfire suppression. The problem features a combinatorial resource allocation structure with endogenous wildfire demand and non-linear wildfire dynamics. We formulate an integer optimization model in a time-space-rest network representation of crew assignments and a time-state network representation of wildfire dynamics, with linking constraints to ensure consistency between both. We develop a branch-and-price-and-cut algorithm based on: (i) a two-sided column generation scheme that generates fire suppression plans and crew routes iteratively; (ii) a new family of cuts exploiting the knapsack structure of the linking constraints; and (iii) novel branching rules to accommodate non-linear wildfire dynamics. We propose a data-driven approach with double machine learning to estimate wildfire spread as a function of covariate information and suppression efforts, while controlling for the endogeneity of treatment and outcome variables. Extensive computational experiments show that the optimization algorithm scales to practical and otherwise intractable instances; and that the methodology can enhance suppression effectiveness, yielding an additional 9.7 percentage-point reduction total area burned over a wildfire season relative to a greedy baseline.

Keywords: *Wildfires, integer optimization, branch-and-price-and-cut, machine learning, causal inference*

Optimization Models and Methods for Wildfire Asset Protection

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Abstract

This thesis addresses a variant of the Wildfire Asset Protection Problem (WAPP) arising in escaped wildfires, where direct suppression is no longer feasible and available resources must be routed and scheduled to protect threatened assets before fire impact. We study a mathematical optimization model and corresponding solution methods for deciding which assets to protect, which mitigation strategy to apply at each selected asset, and how to route a heterogeneous fleet under operational and safety constraints. The proposed formulation extends the existing WAPP literature by combining capability-based cooperative service with multiple mitigation strategies per asset, explicit time windows on both assets and road arcs, and a reward structure that depends on asset worth and strategy effectiveness under predicted fire conditions. In addition, firefighter safety is incorporated directly through a second objective that maximises the time buffer between mitigation completion and forecast fire arrival. The work is currently focused on the development of the mathematical formulation and on designing suitable exact and heuristic solution approaches for this bi-objective optimisation problem.

Keywords: *Wildfire management, Wildfire Asset Protection Problem, Cooperative routing, Bi-objective optimization*

References

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Acknowledgments: HURRICANE Project: Funded by the European Union Horizon Europe Research and Innovation Program under Grant Agreement N°101168017.

Optimal dispatch of firefighting aircraft in wildfire management

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Abstract

This work presents an optimization model, inspired by the “minimum envy problems” framework, to determine the optimal allocation of firefighting aircraft among active wildfires within a given region. The proposed allocation model accounts for the risk level associated with each wildfire – based on factors such as size and location – as well as the water-carrying capacities of the aircraft. It also incorporates the need to maintain a reserve fleet, ensuring that a certain number of aircraft remain unassigned in order to respond rapidly to newly emerging wildfires.

The developed model was implemented in Python and solved using Gurobi on a set of 75 instances of varying sizes, designed to reflect realistic scenarios. However, for a significant subset of instances, optimal or near-optimal solutions could not be obtained within the strict time limits imposed by the emergency context. To address this limitation, metaheuristic-based algorithms were developed, enabling the computation of high-quality solutions within short timeframes using real-world data.

Keywords: *Optimal allocation, wildfire management, minimum envy, metaheuristic algorithms*

References

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Distributionally robust optimization for wildfire response preparedness

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Abstract

A major factor in avoiding large-scale fires is the success of the initial attack. This success depends on how quickly firefighting resources start attacking fire.

In this work, we propose a two-stage Distributionally Robust Optimisation model to improve the effectiveness of the initial attack by the positioning of firefighting resources taking into account the uncertainty of ignition locations and wind direction. In a first stage the position of the resources is decided. In a second stage, after the ignition(s) location(s) and wind direction are known, the attack positions are decided.

The approach relies on the definition of a set of scenarios and on an ambiguity set based on the Wasserstein metric. A mixed integer programming model is solved by an exact decomposition algorithm that generates second-stage variables and constraints only for scenarios with positive probability, significantly improving the scalability of the instances solved. We report on computational results in instances derived from a actual landscape.

Keywords: *Wildfire suppression; Distributionally robust optimisation; Two-stages; Decomposition algorithm*

Acknowledgments: This work was partially funded by the National Innovation Agency, Portugal 2030, Innovation and Digital Transition Programme, Compete 2030 within the scope of project Firesys (AI-empowered Decision Support System for Wildfire Management (project 21378)).

Efficient Recursive Mode Estimation with Stochastic Approximation

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Abstract

The mean, median, and mode are the three most commonly used measures of central tendency in datasets. Sometimes, the mode provides deeper insights into the underlying structure of the data. Mode estimation is one of the statistical problems with key applications in areas such as statistics, modal clustering, modal bandits, modal regression, and molecular clock analysis. As a result, researchers have continually revisited the challenges of mode estimation and inference [1]. When the underlying probability density function is known, determining the mode becomes a deterministic optimization problem and can be solved either analytically or using numerical techniques such as the Newton method or gradient ascent method. However, without explicit knowledge of the density function, finding the mode is an interesting problem. This paper is organized in that way to approximate the mode of an unknown density function using suitable estimation techniques based on the available sample data. In this paper, we propose an enhanced stochastic ascent algorithm for estimating the mode of an unknown multivariate density function, along with a companion algorithm for estimating the size of the mode, where size of the mode is defined as $\mu = f(\theta)$ with mode θ and density function f . The almost sure convergence analysis is based on the stochastic approximation techniques developed in the literature. Under mild assumptions, we establish the almost sure rate of convergence of the proposed algorithm. Finally, we conduct numerical experiments to assess the algorithm's performance, demonstrating its superior efficiency compared to existing methods.

Keywords: *Mode Estimation, Stochastic Approximation, Stochastic Optimization*

References

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Latent t Models for Ordinal Data: Likelihood Inference beyond the Multivariate Normal

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Abstract

The multivariate normal distribution is commonly regarded as the gold standard latent variable model for correlated ordinal data. In this work, we consider the multivariate Student's t distribution as the underlying latent variable, which introduces additional flexibility through the degrees-of-freedom parameter while preserving the threshold-based construction of ordinal responses. The resulting model, however, poses substantial theoretical and computational challenges for likelihood-based inference. These challenges arise both from the discretization mechanism—requiring repeated evaluation of multivariate t copula probabilities over threshold-defined regions—and from the presence of the degrees-of-freedom parameter itself, which complicates optimization and inference. We discuss how this framework can improve the modeling of dependence in ordinal data and provide practical guidance for conducting maximum-likelihood estimation in this setting.

Keywords: *Discretization, Latent variable, Multivariate Student's t distribution, Ordinal variables, Parameter estimation*

References

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Robust bi-objective mean-CVaR portfolio selection: applications to energy sector

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Abstract

A new approach to optimizing or hedging a portfolio of financial positions is presented and tested with applications to energy market. Motivated by uncertainty in the estimation of problem data we consider robust bi-objective optimization problems with mean and conditional value-at-risk objective functions where the underlying probability distribution of portfolio return is only known to belong to a certain set. To tackle the problem of uncertainty we consider two different approaches: in the first one, uncertainty is represented by an elliptic set centered at the sample estimators of mean and covariance matrix; in the second one, uncertainty takes into account experts beliefs. For both approaches, we derive analytical semi-closed-form solutions for the worst case mean-CVaR portfolio; in addition, we provide a characterization of the location of the robust Pareto frontier with respect to the corresponding original Pareto frontier.

Keywords: *Robustness, Multi-objective portfolio selection, Conditional Value-at-Risk, Efficient frontier*

The effect of asymmetric risk measures in portfolio optimization

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Abstract

This study evaluates the performance of a range of portfolios constructed through the optimization of asymmetric risk measures, including skewness, semi-variance, CVaR, lower partial moments, balanced partial moments, and several related variants. These approaches are benchmarked against traditional strategies such as the minimum variance portfolio and the equally weighted portfolio.

The empirical analysis relies on real data from the US market, with performance assessed over an out-of-sample period. Multiple financial indicators are used to evaluate the models through the portfolio return series, discounting transaction costs, including net return, standard deviation, semi-variance, Sharpe and Sortino ratios, maximum drawdown, and win rate, among others.

Initial findings indicate that portfolios based on asymmetric risk measures tend to outperform their traditional counterparts, showing improvements across various metrics such as net return, Sharpe and Sortino ratios, and win rate.

Keywords: *Portfolio optimization, asymmetric risk measures, out-of-sample performance*

Acknowledgments: The work presented in this paper was partially carried out in the scope of the RiskBigData project: Complex Risk Management in the Big Data Regime (PTDC/MAT-APL/1286/2021), financed by the Portuguese Foundation for Science and Technology (FCT) and the Centre for Mathematics of the University of Coimbra (CMUC, <https://doi.org/10.54499/UID/00324/2025>) under the Portuguese Foundation for Science and Technology (FCT), Grants UID/00324/2025 and UID/PRR/00324/2025.

Physician Scheduling Problem with Emphasis on Workload Balancing

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Abstract

Physician scheduling is a critical operational problem in healthcare, particularly in emergency departments where demand variability and continuous staffing requirements place substantial pressure on clinicians. While existing approaches primarily focus on maximising patient throughput or balancing shift counts, they often overlook the uneven distribution of workload intensity across physicians, contributing to burnout and reduced care quality. This paper proposes an equity-first scheduling framework that explicitly accounts for the interaction between physician capacity and patient demand. We introduce a demand-normalised utilisation measure to quantify workload, capturing clinical intensity rather than time worked, and formulate the scheduling problem as a min-max equity optimisation model with patient coverage as a secondary performance indicator. The problem is addressed both an exact Integer Linear Programming formulation and an enhanced Genetic Algorithm incorporating domain-specific local search and feasibility repair mechanisms. Experiments based on data calibrated from a real emergency department show that the proposed approach reduces workload dispersion by a factor of 3.7–3.9 while incurring only a marginal coverage loss of at most 52 patients per month (+2.4%) compared to throughput-maximising schedules.

Keywords: *Physician scheduling, workload equity, emergency department, genetic algorithm, local search, integer linear programming*

Optimal Selection and Management of Reliable and Resilient Teams Based on Multifunctional Personnel

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Abstract

Reliable IT project team formation can be posed as a discrete optimization problem with role-coverage, reliability, and resilience constraints. We present OptiCrew, which models a team as a multifunctional reconfigurable system. Each candidate is described by a role-skill vector; the competency matrix $A = (a_{ij})$ quantifies the ability of member i to perform role j . Team selection and role assignment are solved via integer programming with multi-criteria objectives: (i) maximize expected project performance, (ii) maximize structural reliability computed by logical-probabilistic models and reliability polynomials, and (iii) maximize flexibility through redundancy and substitutability metrics. Robustness is evaluated under stochastic unavailability and performance-degradation scenarios with optimal role reallocation after failures. Qualitative behavioral evidence is transformed into quantitative variables using AI/NLP and integrated for skill-gap detection and adaptation recommendations. Experiments on realistic scenarios indicate that higher multifunctionality yields higher reliability and lower expected downtime.

Keywords: *Optimization Models, Logical-Probabilistic Modeling, Structural Reliability, Multifunctional Personnel, Team Resilience*

Acknowledgments: The work was supported by the Shota Rustaveli National Science Foundation of Georgia. Grant #SC-24-753.

Scheduling crew reserve duties with flexible times

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Abstract

The crew reserve duty scheduling problem focuses on the tactical planning of reserve duties months prior to the day of operation. These duties designate periods during which crew members remain on-call at operational bases, providing the necessary capacity for dispatchers to mitigate real-time disruptions. The complexity of this optimization problem stems from the high degree of stochasticity regarding the timing and location of demand, which exhibits significant seasonal and spatial variance.

We address a new variant of this problem where reserve duties have a flexible time window that will be fixed closer to the day of operation. The fixed duties can be different for different days according to what is more likely to be the reserve needs on those days. To solve this problem, we propose a hybrid solution framework that integrates a constructive heuristic with a probabilistic demand model.

Our research specifically examines the trade-off between flexibility and schedule robustness. We evaluate the proposed approach using large-scale, nationwide data from a major Northern European passenger railway operator. Through Monte Carlo simulations, we compare solutions across varying degrees of allowed flexibility. The results quantify the impact of temporal adaptability on the ability to cover disruptions, providing insights into how flexible reserve scheduling can enhance operational resilience in complex transport networks.

Keywords: *Crew Reserve Duty Scheduling, Flexible Scheduling, Robustness Analysis, Railway Operations*

Acknowledgments: This research was funded by European Regional Development Fund under project LISBOA2030-FEDER-01057000 (project 17222)

Producing Cost-effective and Robust Schedules for Train Drivers and Guards with Optimization and Stochastic Simulation

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Abstract

Railway operational delays significantly degrade passenger satisfaction and increase operational costs. While crew schedules are traditionally optimized for cost-efficiency, they often lack the resilience to prevent delay propagation. We propose a novel solution method that integrates a stochastic railway operation simulation model with a crew scheduling optimizer through a feedback loop mechanism.

The core of our approach lies in dynamically guiding the optimizer to balance cost-efficiency with robustness. By utilizing statistical data from the simulation — which incorporates real-world delay probabilities from current and future timetables — the optimizer adaptively adjusts time buffers on crew connections prone to delay propagation.

Our framework is designed to handle multi-type crew instances (i.e., drivers and guards) either in parallel or sequentially. For sequential execution, we implement a feed-forward protocol that encourages sharing of connections by favouring the choice of connections selected in the previous optimization run.

We validated the model using nation-wide problem instances provided by Nederlandse Spoorwegen (NS). Experimental results demonstrate that this simulation-optimization approach produces schedules that outperform current NS practices, yielding measurable improvements in both cost-effectiveness and operational robustness. This research highlights the efficacy of integrating stochastic simulation into the optimization loop to manage the inherent uncertainty of large-scale transit networks.

Keywords: *Railway Crew Scheduling, Feedback loop, Operational Robustness, Stochastic Simulation*

Acknowledgments: Co-funded by the European Union under grant HORIZON-ER-JU-2022-FA1-TT-01 (Europe’s Rail FP1-MOTIONAL).

MA5: Large Scale Constrained Optimization Problems

Chair: Nataša Krejic

Room: Santander (Floor 3)

Organizer: Nataša Krejic

A general merit function-based global convergent framework for nonlinear optimization

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Abstract

In this talk, we revisit the convergence theory of the inexact restoration paradigm for nonlinear optimization. The work first identifies the basic elements of a globally convergent method based on merit functions. Then, the inexact restoration method that employs a two-phase iteration is introduced as a special case. A specific implementation is presented that is supported in the solution of regularized subproblems. The proposed inexact restoration method includes more freedom in the computation of iterates and a novel procedure that integrates the computation of the penalty parameter with the optimization phase. It also includes an acceleration step based on solving a classic quadratic programming subproblem. The proposed theoretical framework is broad and flexible enough to provide a convergence theory for inexact restoration applications tailored to specific problems. Theoretical results include asymptotic convergence theory as well as a worst-case iteration and evaluation complexity analysis of the introduced methods. The work concludes with numerical experiments that illustrate the practical performance of two variations of the proposed method for solving general nonlinear programming problems.

Keywords: *General convergence framework, Inexact Restoration, Sequential Quadratic Programming, Complexity, Numerical experiments.*

Acknowledgments: This work has been partially supported by FAPESP (grants 2013/07375-0, 2020/12455-6, 2021/14011-0, 2022/05803-3, and 2023/08706-1), CNPq (grants 302073/2022-1, 311830/2023-4, and 407147/2023-3), and CAPES.

CLASP: Online Learning algorithms for convex losses and squared penalties

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Abstract

In this talk, we address Constrained Online Convex Optimization, where a learner chooses actions iteratively, observes both unanticipated convex loss and convex constraint, and accumulates loss while incurring penalties for constraint violations. We present CLASP, a framework that minimizes cumulative loss together with squared constraint violations. Our theoretical analysis departs from prior work by fully leveraging the variety of cutter operators, and contraction properties such as the strongly quasi-nonexpansiveness, a proof strategy not previously applied in this setting. For convex losses, we show that CLASP algorithms achieve regret $O(T^{\max\{\beta, 1-\beta\}})$ and cumulative squared penalty $O(T^{1-\beta})$, for any $\beta \in (0, 1)$. Most importantly, for strongly convex problems, we provide the first logarithmic guarantees on both regret and cumulative squared penalty: CLASP algorithms guarantee $O(\log T)$ on both regret and cumulative squared penalty.

Keywords: *Regrets bounds, Constrained Online Convex Optimization, Strong Convexity, Strongly Quasi-Nonexpansive Operators*

Acknowledgments: This work has been supported by FCT through LARSyS funding (DOI: 10.54499/LA/P/0083/2020, 10.54499/UIDP/50009/2020 and 10.54499/UIDB/50009/2020), and by NOVA LINCS (UID/04516/2025) with the financial support of FCT.IP (DOI: 10.54499/UID/04516/2025). This research was carried out under Project “Artificial Intelligence Fights Space Debris” No C626449889-0046305 co-funded by Recovery and Resilience Plan and NextGeneration EU Funds, www.recuperarportugal.gov.pt.

Ipas: An Adaptive Sample Size Method for Weighted Finite Sum Problems with Linear Equality Constraints

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Abstract

Optimization problems with the objective function in the form of weighted sum and linear equality constraints are considered. Given that the number of local cost functions can be large as well as the number of constraints, a stochastic optimization method is proposed. The method belongs to the class of variable sample size first order methods, where the sample size is adaptive and governed by the additional sampling technique earlier proposed in unconstrained optimization framework. The resulting algorithm may be a mini-batch method, increasing sample size method, or even deterministic in a sense that it eventually reaches the full sample size, depending on the problem and similarity of the local cost functions. Regarding the constraints, the method uses controlled, but inexact projections on the feasible set, yielding possibly infeasible iterates. Almost sure convergence is proved under some standard assumptions for the stochastic framework, without imposing the convexity. Numerical results on relevant problems from CUTEst collection and real-world data sets for logistic regression show the stability and the efficiency of the proposed method when compared to the state-of-the-art methods.

Keywords: *Linear equality constraints, finite sum problems, stochastic methods, additional sampling*

Acknowledgments: This work is supported by the Science Funds of Republic of Serbia, project LASCADO, grant no. 7359

MA6: Applications and Algorithms for Bilevel Optimization

Chair: Margarida Carvalho

Room: Staples (Floor 3)

Organizer: Margarida Carvalho

A hybrid PSO–MILP approach for bilevel multi-follower optimization of demand-side flexibility aggregation

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Abstract

The growing penetration of variable renewable generation calls for enhanced demand-side flexibility to maintain power system reliability. Flexibility aggregators are key enablers coordinating residential consumers/prosumers and trading aggregated flexibility in local markets. This work proposes a bilevel (BL) multi-follower mixed-integer optimization model that captures the hierarchical interaction between an aggregator and multiple heterogeneous households. At the upper level, the aggregator sets flexibility rewards to maximize profit, defined as the revenue obtained from the grid operator minus the penalties for flexibility shortfalls and the rewards paid to consumers. At the lower level, each consumer optimizes the scheduling of home energy resources — including shiftable appliances, thermostatically controlled loads, electric vehicles, stationary batteries and microgeneration — in response to rewards and energy prices to minimize costs while respecting comfort constraints. To solve this challenging BL optimization problem, we develop a hybrid approach combining particle swarm optimization (PSO) for the upper-level problem with an exact mixed-integer linear programming (MILP) solver for the lower-level subproblems. Computational experiments using real data for a 24-hour horizon with 15-minute resolution across eight consumer profiles show the efficiency and robustness of the proposed PSO–MILP framework under constrained computational budgets.

Keywords: *Bilevel multi-follower optimization, Demand-side flexibility aggregators, Hybrid metaheuristics, Particle swarm optimization, Mixed-integer linear programming*

A Bilevel Optimization Framework for Incentive Design in One-Way Car-Sharing Systems

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Abstract

Municipalities play a key role in promoting sustainable urban mobility by reducing congestion and emissions, and ensuring equitable access to transport services. Car-sharing systems are a promising solution to achieve these goals, improving asset utilisation and offering flexible mobility. However, without appropriate intervention, private operators may prioritise profit-driven decisions that do not guarantee adequate spatial coverage or accessibility.

To address this, municipalities can design public policies to align private decisions with social objectives. Meanwhile, operators must integrate strategic, tactical and operational decisions, namely station placement and capacity, fleet size and pricing. The problem becomes more complex when users can pick up a vehicle at one location and drop it off at another (known as one-way car-sharing), leading to imbalances in vehicle stock across stations.

This interaction creates a hierarchical structure in which municipalities act as leaders by defining incentive schemes, while private operators act as followers, responding with profit-maximising decisions. We model this problem as a bilevel optimisation program. In the upper-level problem, the municipality determines the incentives to provide with the goal of maximising a gravitational accessibility function that captures system attractiveness and spatial coverage. The incentives can be in the form of lump-sum payments, parking space subsidies, and per-trip subsidies, subject to a limited budget. The lower-level problem determines which stations to open, along with their capacities, fleet sizes, and relocation staffing, aiming to maximise profit under operational constraints.

Keywords: *car-sharing, bilevel optimisation, public policies*

Embedded dynamic programming for bilevel pricing problems

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Abstract

Pricing problems are bilevel optimization models in which a leader sets prices for a subset of items (the tolled items), while a follower solves an optimization problem parametrized by these pricing decisions. The leader's objective is to maximize revenue based on the tolled items chosen by the follower. In this talk, we begin with the classic Network Pricing Problem to illustrate the key components of the dualize-and-combine methodology for solving bilevel problems. We then focus on Combinatorial Pricing Problems, emphasizing the dualization process within this framework. To make the methodology applicable, we introduce dynamic programming models for the follower's problem, notably decision diagrams and the novel selection diagrams. Given that our dualize-and-combine reformulation is non-compact, we propose a cutting-plane method to solve Combinatorial Pricing Problems efficiently in practice. Finally, we present insights from computational experiments to evaluate our approach.

Keywords: *Bilevel Optimization, Duality, Combinatorial Optimization, Cutting-plane*

Acknowledgments: This work was funded by FRQ-IVADO Research Chair in Data Science for Combinatorial Game Theory, and the NSERC grant 2019-04557 and 2024-04051. This research was enabled in part by support provided by Calcul Québec and the Digital Research Alliance of Canada.

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Title: Advanced Hybrid Quantum Optimization Frameworks

Chair: Paula Amaral

Room: CGD (Floor 2)

Abstract

This talk introduces quantum optimization methods, focusing on the ground state search problem of the Ising Hamiltonian via Variational Quantum Algorithms (VQAs). To enhance the performance and reliability of VQAs in solving binary linear programming problems, we propose and evaluate two novel operational frameworks exploiting the structure of linear programs and relevant aspects of their physical encodings in a quantum computer.

Specifically, we show classically integrated hybrid quantum optimization algorithms that utilize these aspects in two different ways: a constraint generation routine, which aims to provide high-quality solutions quickly, and a complete branch-and-bound scheme that decomposes the original problem with respect to the physical structure and is measurable by metrics like the primal-dual integral.

MB1: Routing in Healthcare**Chair:** Pedro Martins**Room:** Amphit. 1 (Floor 4)

A Matheuristic for the Traveling Salesman Problem with positional consistency

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Abstract

In addition to the difficulties directly related to the Covid19 pandemic, regular healthcare services faced challenges during lockdown times. Permanence in the healthcare facilities was discouraged to avoid contagion, which added constraints to the scheduling of appointments in health centers. Inspired by the Portuguese Health System's approach of dividing the healthcare professionals' shifts into homogeneous time slots, the Traveling Salesman Problem with positional consistency constraints (CTSP) uses routes to represent work shifts for the day, relative positions to represent the time slots and consistency constraints to model that patients that need to be seen by several healthcare professionals should be assigned to the same time slot for all of them.

This work proposes a matheuristic approach for the CTSP, which consists of an iterated local search algorithm that uses local search to diversify the exploration of the solution space and a restriction version of the CTSP, where the complexity is significantly reduced by fixing the positions of the consistent nodes, to significantly improve the quality of the local optima found by the local search.

Computational results show that, for instances 48-76 nodes and 5 routes, or 96-171 nodes and 10 routes, the matheuristic obtains, in short computational times, significantly better solutions than an exact method in 10 hours, obtaining optimal or near optimal solutions for instances where the optimal solution is known.

Keywords: *combinatorial optimization, traveling salesman problem, positional consistency, iterated local search*

Acknowledgments: This research was supported by Portuguese National Funding from FCT – Fundação para a Ciência e a Tecnologia under project UID/04561/2025 and grant SFRH/BD/146812/2019

Decomposition algorithms for an Integrated Home Health Care Planning Problem

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Abstract

The growing demand for home-based care, driven by an aging population and chronic diseases, increases pressure on providers, making effective service delivery design essential to ensure high-quality care while controlling costs. In this work, we address an integrated home health care planning problem that jointly considers assignment, scheduling, routing, and rostering decisions in the medium term. The proposed mixed-integer formulation incorporates time window constraints, operator synchronization, labor, and skill compatibility. The objective function minimizes routing, overtime, and idle-time costs while penalizing unserved requests. To address the problem complexity, we propose alternative cluster-based decomposition methods that efficiently exploit the problem's structure. Computational experiments on real-world instances demonstrate that the proposed methods yield high-quality solutions and significantly improve scalability compared to the monolithic formulation.

Keywords: *Home Health Care, Clustering, Decomposition Algorithms, Mixed-Integer Programming*

Intertwined vehicle routing problem for home healthcare services

Pedro Martins¹

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Abstract

Given a simple oriented and centralized graph $G = (N, A)$, with weights on the nodes and on the arcs, where the central node represents the depot and the other nodes represent customers, the Intertwined Vehicle Routing Problem (IVRP) involves the construction of two types of routes for visiting the customers. The routes of the first type cover all customers (N) while the routes of the second type cover just a subset ($N' \subseteq N$). This means that each customer in N' should be visited by two routes, one of each type, and these two routes should meet together when visiting this customer. We know the time spent in each customer and the times for traveling between customers. The objective is to minimize the sum of the delays among the arrival instants of the two vehicles to all customers belonging to N' . This work applies the IVRP to home healthcare services, considering that routes of type 1 involve nurses' teams and each route of type 2 has a doctor assigned. All customers in N require the visit of a nurses' team and the subset of customers in N' further require the visit of a doctor. The central node is the hospital. The motivation of this study is to plan daily routes for visiting home healthcare outpatients or patients requiring post-operative follow-up, creating separate routes for nurses and doctors. We propose formulations and discuss results from a computational standpoint and from an applied perspective. The IVRP can also be applied in other contexts.

Keywords: *vehicle routing problem, intertwined routes, home healthcare routes*

Acknowledgments: This work is partially supported by FCT - Fundação para a Ciência e a Tecnologia, I.P., through national funds, under the project UID/04561/2025. The work is also partially supported by Coimbra Business School - ISCAC, Polytechnic Institute of Coimbra.

MB2: Optimization for Machine Learning

Chair: Paula Amaral

Room: Amphit. 2 (Floor 4)

Organizer: Paula Amaral

Anomaly Detection and Root Cause Analysis of Retainability Contextual Anomalies in Radio Access Networks

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Abstract

The increasing complexity of mobile communication networks, driven by the rapid growth in connected devices and data traffic, has made network monitoring and troubleshooting more challenging for Mobile Network Operators (MNOs). Traditional approaches to fault diagnosis rely heavily on manual analysis, which is time-consuming, error-prone, and difficult to scale.

In this work, we propose an automated framework for anomaly detection and root cause analysis in Radio Access Networks (RANs) using Machine Learning (ML) techniques. The approach combines unsupervised anomaly detection methods with explainability tools to identify abnormal patterns in Key Performance Indicators (KPIs) and uncover their underlying causes. Dimensionality reduction and clustering techniques are used to structure the data, while model interpretability methods such as Shapley Additive Explanations (SHAP) provide insights into feature importance and decision-making processes.

Experimental results demonstrate that the proposed methodology effectively detects anomalous behavior and assists in identifying its root causes, significantly reducing the time required for network troubleshooting. This contributes to improved operational efficiency and more reliable network performance.

Keywords: *Root Cause Analysis, Machine Learning, Radio Access Networks, SHAP*

Acknowledgments: This work is funded by national funds through the FCT - Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UID/00297/2025 (doi.org/10.54499/UID/00297/2025) and UID/PRR/00297/2025 (doi.org/10.54499/UID/PRR/00297/2025) (Center for Mathematics and Applications - NOVA Math). This work was also supported by the European Union via the European Regional Development Fund (ERDF) under the COMPETE 2030 Programme [COMPETE2030-FEDER-01176700].

A classification method based on a cloud of spheres

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Abstract

In [1] and [2], we develop a binary classification model to distinguish a specific class that we intend to identify (*e.g.*, fraud, spam, disease), through a *cloud of hyperspheres* that circumscribes the points of such target class. It is intended to build a model based on a single cloud and not on a disjoint set of clouds, establishing this condition on the connectivity of a graph induced by the hyperspheres. To solve the problem, designed by a Cloud of Connected Hyperspheres, a quadratic formulation with continuous and binary variables (MINLP) is proposed with the minimisation of the number of hyperspheres. The issue of connectivity implies in many models the imposition of an exponential number of constraints. However, because of the specific conditions of the problem under study, connectivity is enforced with linear constraints that scale quadratically with K , which serves as an upper bound on the number of hyperspheres. This classification model is effective when the structure of the class to be identified is highly non-linear and non-convex, also adapting to the case of linear separation. Unlike neural networks, this classification model is transparent, with the structure perfectly identified. No kernel functions are used and it is not necessary to use meta-parameters, unless it is intended also to maximise the separation margin. Finding the global optima for large instances is quite challenging, and to address this, heuristics are proposed.

Keywords: *Supervised Classification, Geometric Classifier, Interpretability*

References

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Acknowledgments: This work is funded by national funds through the FCT – Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UID/00297/2025 (<https://doi.org/10.54499/UID/00297/2025>) and UID/PRR/00297/2025 (<https://doi.org/10.54499/UID/PRR/00297/2025>) (Center for Mathematics and Applications – NOVA Math).

Non Linear Support Vector Machine for Interval-valued Data

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Abstract

In this talk we present a generalization of the Spherical Support Vector Machine method, in which the separator is a sphere, applied to Interval-value data. This type of data belongs to a more general class, known as Symbolic Data, for which features are described by sets, intervals or histograms instead of classic arrays. This paradigm is raising interests in our days, specially in the context of Big Data. On the other end, SVM is a classic and well studied method for classification, and in the classical approach the separation is defined by an hyperplane. Generalizations of this concept have merged, with the use of non-linearity defined by Kernel functions. As an alternative to Kernell functions some authors have proposed non linear separation functions, with the advantage of keeping the classification in the feature space. In particular spherical separations have been used and in this paper we explore this idea for interval data. We also present a relaxation model and the performance of these two formulations for classification purpose is tested against standard methods.

Keywords: *SVM, Interval-valued Data, Symbolic Data, Spherical separation*

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Environmental Impact Optimization of the Soy Supply Chain: A Case Study of the Mexican Food Restaurant Industry

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Abstract

This study evaluates the environmental footprint of distributing plant-based proteins for a company with multiple restaurant chains across a complex, multi-echelon Mexican cold chain network. Because continuous cold chain management represents a high-impact logistical stage where energy-intensive requirements can compromise sustainability goals, this research utilizes a comparative analysis to quantify the impact of three distinct sourcing scenarios with varying thermal profiles: S1) Edamame imported from China (maritime freight, frozen chain), S2) Tofu imported from the U.S. (road freight, dry chain), and S3) a hypothetical domestic Textured Vegetable Protein scenario (domestic road, dry chain).

To identify the optimal, most sustainable product flow, a Linear Programming optimization model was developed and integrated with Life Cycle Assessment (LCA)-derived emission factors. The model's objective function minimized the total domestic logistical CO₂eq emissions, subject to a fixed network structure and core constraints including supply capacity, fixed customer demand, and intermediate storage capacity at Distribution Centers and Regional Warehouses.

The optimization results revealed that refrigerated long-haul road transport from the U.S. border (Scenario S2) is the primary logistical emission hotspot, generating the highest per-kilogram domestic CO₂eq impact. However, the full cradle-to-gate LCA functional unit of 100 grams of protein, confirmed that the product's protein concentration is a relevant environmental factor just as the transportation and refrigeration infrastructure.

Keywords: *Optimization, Logistics, Supply chain, Sustainability*

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Joint Optimization of Schedule and Inventory Replenishment Policy

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Abstract

In supply chain management, shipments from multiple suppliers to a central warehouse are often coordinated, taking advantage of lower transportation fees for full truckloads. This is a form of the classical Joint Replenishment Problem (JRP) under stochastic demand. However, as networks grow and the number of inbound deliveries increases, the warehouse's handling capacity remains limited, a potential bottleneck that can disrupt the supply chain. A warehouse replenishment schedule is therefore essential. Two mutually dependent yet contradicting decisions must be made in this setting: a static warehouse delivery schedule and a dynamic JRP inventory policy. Minimizing overall costs involves balancing schedule costs, expected transportation costs, and inventory-related expenses while adhering to warehouse handling limitations. We first formulate the problem as an exact bilevel program that jointly determines both decisions. To circumvent the curse of dimensionality, we propose a MILP adaptation that determines period-dependent can-order policy parameters. To the best of our knowledge, this is the first non-iterative method to determine can-order parameters, which helps explain its scalability. We benchmark it against a tailored Genetic Algorithm. Our results show that the MILP consistently finds less costly solutions under the same running time.

Keywords: *Inventory Control, Joint Replenishment Problem, Bilevel Programming, MILP, Genetic Algorithm.*

Multi-period supply chain network design with disruption risk: A bi-objective stochastic approach

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Abstract

We propose a novel bi-objective two-stage stochastic program for designing resilient and agile three-echelon supply chain networks. It integrates conditional backup sourcing, facility fortification, bounded delivery delays, and minimum service levels under demand, cost and disruption uncertainty. Strategic first-stage decisions include facility deployment and supplier choice; second-stage decisions cover product flows across the network and trigger backup supplier activation. Minimizing expected cost and unmet demand, a tailored MIP-based heuristic, embedded within an epsilon-constraint framework, efficiently solves large instances and reveals critical cost-resilience-responsiveness trade-offs to support decision-making.

Keywords: *Resilient and flexible supply chains, Network design, Two-stage stochastic programming,*

References

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Acknowledgments: This work is funded by national funds through the FCT – Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UID/00297/2025 (<https://doi.org/10.54499/UID/297/2025>) and UID/PRR/297/2025 (<https://doi.org/10.54499/UID/PRR/297/2025>) (Center for Mathematics and Applications – NOVA Math).

MB4: Derivative-Free Optimization I

Chair: Everton Silva

Room: Santander (Floor 3)

Organizer: Everton Silva & Yiwen Chen

A Finite-Difference Trust-Region Method for Convexly Constrained Smooth Optimization

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Abstract

We propose a derivative-free trust-region method based on finite-difference gradient approximations for smooth optimization problems with convex constraints. The proposed method does not require computing an approximate stationarity measure. For nonconvex problems, we establish a worst-case complexity bound of $\mathcal{O}\left(n\left(\frac{L}{\sigma}\epsilon\right)^{-2}\right)$ function evaluations for the method to reach an $(\frac{L}{\sigma}\epsilon)$ -approximate stationary point, where n is the number of variables, L is the Lipschitz constant of the gradient, and σ is a user-defined estimate of L . If the objective function is convex, the complexity to reduce the functional residual below $(L/\sigma)\epsilon$ is shown to be of $\mathcal{O}\left(n\left(\frac{L}{\sigma}\epsilon\right)^{-1}\right)$ function evaluations, while for Polyak-Lojasiewicz functions on unconstrained domains, the bound further improves to $\mathcal{O}\left(n\log\left(\left(\frac{L}{\sigma}\epsilon\right)^{-1}\right)\right)$. Numerical experiments on benchmark problems and a model-fitting application demonstrate the method's efficiency relative to state-of-the-art derivative-free solvers for both unconstrained and bound-constrained problems.

Keywords: *Derivative-Free Optimization, Trust-Region Method, Worst-Case Complexity*

References

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A KKT-Free Augmented Lagrangian Method for Hydraulic Problems

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Abstract

This work presents Augmented Lagrangian methods for constrained optimization without the use of derivatives. Unlike classical approaches, the framework does not rely on the existence of derivatives or on KKT conditions to establish convergence results. The theoretical analysis is developed under general assumptions, allowing the method to be applied to problems where the objective function and the constraints may be non-differentiable. A flexible formulation is considered in which different derivative-free algorithms can be used to approximately solve the subproblems that arise during the optimization process.

Numerical experiments are presented in the context of river-flow modeling, where the goal is to estimate hydraulic parameters by minimizing their dispersion while satisfying differential equations and empirical observations. This calibration problem is formulated as a constrained optimization problem and solved using the proposed approach.

The application is related to the CRIAB research project, which develops mathematical models for natural-disaster prevention, particularly in the context of dam failures.

Keywords: *River flow modeling, Augmented Lagrange, Derivate-free optimization*

References

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Acknowledgments: The author acknowledges the support of IMECC – Universidade Estadual de Campinas (UNICAMP) and CNPq (National Council for Scientific and Technological Development).

Nonlinear Derivative-free Constrained Optimization with a Penalty-Interior Point Method and Direct Search

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Abstract

SID-PSM is a well-established directional direct search method that employs quadratic polynomial models to enhance numerical efficiency in both the search and poll steps. However, constraint handling is based on an extreme barrier approach, which does not exploit information regarding constraint violations. In this work, we propose a hybrid framework that combines a mixed penalty-interior point strategy with direct search to address nonlinear constrained derivative-free optimization problems. The approach is based on a merit function in which the set of nonlinear inequality constraints is partitioned into two groups: one handled with a logarithmic barrier term, and the other, together with equality constraints, treated via penalization. This strategy is integrated into SID-PSM, enabling the effective treatment of general nonlinear constraints. Convergence to Karush-Kuhn-Tucker (KKT) stationary points is established under standard continuous differentiability assumptions, without requiring convexity. Numerical experiments on analytical test problems and an engineering application demonstrate the robustness, efficiency, and overall effectiveness of the proposed method when compared to state-of-the-art solvers.

Keywords: *Derivative-free optimization, Constrained optimization, Interior point methods, Direct search, Polynomial interpolation*

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Forecast Adjustment in Multicriteria Analysis Using Parameterisable Functions

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Abstract

This investigation proposes improvements to the TODIM multicriteria decision-support method. A new parametrisation of the TODIM phi function is introduced using techniques common in binary-response statistical models, aiming to increase flexibility and better reflect real decision-maker behaviour.

Empirical validation uses preference and weighting data from 20 higher-education students (Leoneti, 2016). Results from the original TODIM are compared with an adapted version (TODIMAdp). Spearman correlation with participants' initial rankings shows that TODIMAdp performs better on average (0.29 vs. 0.20) and correctly predicts the top-ranked alternative in 50% of cases, compared with 37% for the original method.

Assuming MCDM ranking methods can be evaluated by their ability to reproduce initial preferences, the study finds that TODIMAdp's additional parameters increase modelling flexibility and predictive capacity. The results suggest that further exploration of parameter combinations may strengthen TODIM's descriptive and forecasting performance in applied multicriteria decision contexts.

Keywords: *bounded rationality, prospect theory, TODIMAdp, forecast, parameterization*

References

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Acknowledgments: This work is funded by national funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., under CEAUL Research Unit, UID/00006/2025, DOI: <https://doi.org/10.54499/UID/00006/2025>, and by the European Union – NextGenerationEU through the project UID/PRR/00006/2025, DOI: <https://doi.org/10.54499/UID/PRR/00006/2025>.

Designing Conflict-Free Allocations: A Structural Optimization Approach to Fair Division

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Abstract

The fair division problem deals with allocating resources among multiple parties such that each receives a fair share according to their valuation. We study this problem in a specific setting where resource availability is interval-based rather than fixed. The aim is to find an optimal resource structure within given bounds to eliminate or mitigate conflict between receiving parties. These resources are provided by an authority constrained by unit costs and a strict total budget. We formulate this as a multiobjective optimization problem under various fair division settings, incorporating utilitarian and egalitarian goals, alongside classic envy-freeness conditions. We present computational experiments on synthetic data. Furthermore, we conduct a parametric sensitivity analysis to explore how varying levels of resource flexibility, budget restrictiveness, and preference heterogeneity impact the achievable fairness. Preliminary results show that dynamically restructuring resources within interval bounds achieves an improved fairness level not dominated by any other configuration. Additionally, the analysis highlights budget thresholds where the performance of utilitarian and egalitarian approaches significantly diverges.

Keywords: *Fair division, Multiobjective optimization, System design*

Acknowledgments: The research was supported by Czech Science Foundation Grant No. 25-15714S.

Comparison of epsilon-constraint algorithms for a bi-objective CVRP

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Abstract

This presentation addresses the Bi-objective Capacitated Vehicle Routing Problem. Routing problems are a class of optimization problems with significant real-world applications, often involving conflicting objectives, and are inherently challenging to solve. We conduct a computational study using benchmark instances from the literature to compare the exact epsilon-constraint methods proposed by Mavrotas (2013) [1] and Mesquita-Cunha et al. (2023) [2] with a novel approach that enhances these methods by incorporating warm starts derived from supported non-dominated points and constructive and improvement heuristics.

Keywords: *Multiobjective Optimization, Exact Methods, Optimization Models and Methods, VRP, Pareto-Optimal Frontier*

References

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Acknowledgments: This work is financed by Portuguese funds through the FCT - Foundation for Science and Technology, I.P., under projects LISBOA2030-FEDER-00853500 (SUPPORT-HH) and UID/97/2025 (CEGIST).

MC1: Optimization Methods for Rich Routing Problems**Chair:** Telmo Pinto**Room:** Amphit. 1 (Floor 4)**Organizer:** Cláudio Alves

Exploring New Sequential and Parallel Variable Neighborhood Search Algorithms for the Pollution Routing Problem

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Abstract

In this talk, we address the Pollution Routing Problem, a rich vehicle routing problem where fuel, emissions and driver wages depend on both vehicle speed and load along each arc. We propose several algorithms based on Variable Neighborhood Search (VNS), with original neighborhood structures that modify simultaneously the customer sequences and arc speeds. A key feature of our approach is the integration of speed decisions into route construction and local search, rather than as a post-processing step, allowing the search to exploit the trade-offs between travel time, fuel use and emissions. We develop General VNS, Smart VNS and Smart and Parallel GVNS (SPGVNS) variants, and test them on standard PRP benchmark sets derived from real UK instances, with up to 200 customers. SPGVNS has the best overall performance, often improving on or matching the best known solutions in the literature while using less computing time. Additionally, we developed and evaluated several hybrid CPU–GPU schemes for parallelizing local search in our VNS algorithms for the PRP. We explore different kernel designs and neighborhood evaluation strategies. An extensive experimental study on benchmark instances is presented to illustrate how these parallel approaches perform in practice and to quantify their potential for accelerating advanced metaheuristics.

Keywords: *Pollution Routing Problem, Variable Neighborhood Search, Parallel Computation*

Acknowledgments: This work has been supported by FCT, through national funds from MCTES – Ministério da Ciência, Tecnologia e Ensino Superior, and by European Social Fund through NORTE2020, within the research grant SFRH/BD/146217/2019. This work has also been supported by FCT within the R&D Unit Project Scope UID/00319/Centro ALGORITMI (ALGORITMI/UM) <https://doi.org/10.54499/UID/00319/2025>.

Column Generation Algorithm for the Prisoner Transportation Problem

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Abstract

This paper presents a column generation approach for the prisoner transportation problem (PTP) with multiple depots. The PTP is a variant of the well-known Dial-a-Ride Problem, which consists of assigning prisoner transport requests from pickup points to delivery points to vehicles while respecting several operational requirements, such as time windows, vehicle capacity, ride time, and the assignment of prisoners to multicompartment vehicles. The primary objective of the optimization approach is to minimize the total travel time for all vehicles. Since the PTP is NP-hard, a column generation-based algorithm has been designed, in which the master problem is a set partitioning problem, and the subproblems are formulated as complex resource-constrained shortest-path problems. The subproblems are solved employing dynamic programming and a Variable Neighborhood Search (VNS) algorithm. The CG algorithm was evaluated using benchmark instances from the literature. The dynamic programming variant was more effective for instances with up to 10 requests, while the VNS performed better for instances with more than 50 requests. Our approach, with appropriate time limits, yields solutions that are at least as good as previously reported best-known solutions on 100% of the tested instances, with an overall average improvement of 12.29%, demonstrating the efficacy of the developed solution method.

A Bilevel Approach for Production Routing Problems

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Abstract

The integration of production, inventory, and distribution decisions has become a key topic in supply chain optimisation, progressing from classical vehicle routing models to more comprehensive Production Routing Problems. However, most existing approaches assume a single decision maker overseeing both production and distribution, which does not reflect many real-world settings where manufacturers and distributors operate separately and under different objectives. This study addresses that gap by proposing a bilevel production routing model for a multi-depot, multi-plant context. The upper level captures the distributor's routing and procurement decisions, while the lower level represents production responses. By integrating production, routing, and inventory within a hierarchical framework, the model provides a realistic representation of supply chain interactions and is solved through an exact algorithm.

Keywords: *Bi-level optimization, Integrated production and distribution*

Bi-level Optimization of Integrated Production and Routing under Carbon Pricing

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Abstract

The integration of production planning and distribution routing has become increasingly important, as isolated decisions often lead to suboptimal solutions. At the same time, reducing carbon emissions is now a key concern in both business strategy and public policy. In this talk, we address the Production Pollution Routing Problem (PPRP), which jointly considers operational costs and emissions from production and transport activities. A bi-level formulation for the PPRP is presented. The upper level represents a regulatory authority that defines a carbon tax per kilogram of CO_2 emitted and a subsidy for emission reductions. The lower level represents a company that jointly optimises production scheduling and route planning in order to minimise total costs under the imposed policy. The model combines linear programming at the upper level with mixed-integer programming at the lower level. The model also includes multiple facilities, heterogeneous fleets, and perishable products, increasing its practical relevance. Results show that substantial emission reductions can be achieved without compromising economic viability, and that production decisions may have a stronger impact on total CO_2 reduction than route optimisation alone.

Keywords: *Bi-level optimization, Production routing problem, Carbon pricing*

MC2: Diversity in Network Optimization

Chair: Marta Pascoal

Room: Amphit. 2 (Floor 4)

Organizer: Marta Pascoal & Maria Teresa Godinho

Enhancing the resilience of telecommunication networks through geodiversification

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Abstract

The resilience of telecommunication networks can be seen as the ability to withstand and recover rapidly from disruptions provoked by deliberate attacks, accidents, or naturally occurring threats or incidents. One way to increase the preparedness of a network is to provide alternative paths for communication traffic. When dealing with disasters that can affect large portions of land, this can be achieved by imposing a safety geographical distance between the paths.

Given an integer $K \geq 2$ and two nodes in a network, two related problems are found in the literature: finding K paths linking those nodes, separated by a geographical distance of at least $D > 0$ and with minimum cost; and finding the maximum value of D for which such K paths exist. The utility of the first problem depends on the considered value D , and the best D can be found by solving the second. Therefore, the two problems are complementary and equally important. However, the later has been far less studied than the former.

This talk focuses on these problems, devoting special attention to the second: an integer linear programming (ILP) formulation is introduced for the first problem; two ILP formulations (which are theoretically compared) and a binary search method are proposed for the second.

The proposed methods are tested through a set of computational experiments on reference networks in telecommunications, showing that both problems can be solved efficiently by our methods.

Keywords: *Telecommunication networks resilience, Alternative paths, Geodiversity*

Acknowledgments: The first author acknowledges the Center for Mathematical Studies (CEM.UL) for its support in the completion of this work. The work was also partially financially supported by the Portuguese Foundation for Science and Technology under project grants UID/MAT/00324/2020 and UID/MULTI/00308/2020.

Risk Reduction vs. Fair Cost Allocation: Rethinking Gateway Policies for Hazmat Transport

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Abstract

The authority governing a territory can mitigate the risk associated with the transportation of hazardous materials on a road network by enforcing regulatory policies that influence carriers' route choices between origin and destination.

The risk mitigation policy considered in this study consists of assigning each carrier a mandatory transit node, hereafter referred to as a gateway.

The joint problem of gateway location and assignment naturally leads to a bilevel optimization framework: at the upper level, the authority minimizes a suitable network risk measure by determining both the placement of gateways and their assignment to carriers; at the lower level, each carrier selects a minimum-cost path from its origin to the assigned gateway and from the gateway to its destination.

Despite its effectiveness in reducing overall risk, this policy induces additional costs for carriers relative to their unconstrained shortest paths and may lead to risk concentration at specific nodes of the network.

Building upon previous work, this study investigates the interplay between equity-oriented measures aimed at limiting cost disparities among carriers and the resulting spatial distribution of risk across the network. Moreover, it assesses the impact of such measures on the total network risk and demonstrates how a multi-commodity flow formulation can be extended to capture these features within a unified modeling framework.

Keywords: *hazardous material, gateway, bi-level programming, fairness*

References

- [1] Cappanera, P. and Nonato, M. (2025), Incorporating Fairness Into the Gateway-Based Risk Mitigation Policy for Hazmat Transport. *Networks*, 85: 205-222. <https://doi.org/10.1002/net.22259>

The bi-objective problem of finding K shortest and dissimilar paths: models and algorithm

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Abstract

The resilience of the solution for routing problems may be compromised by alternative solutions that share many network resources. The problem of finding sets of K paths between given origin and destination nodes in a network which minimize a total cost function and maximize the paths dissimilarity intends to tackle this challenge by promoting the search for low cost solutions that simultaneously have little in common [1]. In this talk we revisit bi-objective single-commodity and discretized flow formulations which model the similarity based on two assumptions: the minimization of the extent of the paths overlap, measured in terms of the number of arc reuses, and the minimization of the number of pairwise overlaps in arcs of the paths. An ϵ -constrained method is described for approaching the two versions of the problem [2].

Computational experiments compare the results found by the two approaches in terms of the efficiency regarding computational time and of the dissimilarity of the solutions for each problem.

Keywords: *Alternative paths, Cost, Dissimilarity, Integer linear programming, Bi-objective optimization*

References

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Acknowledgments: Work partially financially supported by the Portuguese Foundation for Science and Technology under project grants UIDB/04561/2020, UID/MAT/00324/2020 and UID/MULTI/00308/2020. The work was also supported by the Center for Mathematical Studies (CEM.UL).

An integer linear programming reformulation for optimizing pairwise overlaps

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Abstract

In this talk we describe a new reformulation technique in the context of optimization problems with the goal of minimizing the number of pairwise overlaps within the solutions. We show that while the models obtained using the new technique do not provide better linear programming bounds than the ones obtained using a well-known reformulation technique from the literature [2], they are faster to solve, especially for difficult problems. A set of computational tests performed on two problems is presented: a K paths problem between two nodes in a network aiming at minimizing the number of pairwise arc overlaps between them [2]; and a set covering problem aiming at minimizing the number of pairwise overlaps between the different sets in the cover [3].

Keywords: *Pairwise overlaps, Paths, Covers, Integer linear programming, Discretized formulations*

References

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- 2 Gouveia, L.: A $2n$ constraint formulation for the capacitated minimal spanning tree problem. Operations Research 43, 130–141 (1995)
- 3 Moghanni, A., Pascoal, M., Godinho, M.T.: Finding K dissimilar paths: singlecommodity and discretized flow formulations. Computers & Operations Research, 147, 105939 (2022)

Acknowledgments: Work partially financially supported by the Portuguese Foundation for Science and Technology under project grants UID/MAT/00324/2020 and UID/MULTI/00308/2020. The work was also supported by the Center for Mathematical Studies (CEM.UL).

MC3: Scheduling I

Chair: Dalila Fontes

Room: Delta (Floor 3)

Organizer: Dalila Fontes

Beyond Myopic Plans: Learning Policy Parameters for MDP-based Workforce Scheduling

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Abstract

Workforce management in retail environments faces constant uncertainty driven by demand fluctuations and employee absences. While Mixed-Integer Programming (MIP) is effective for handling complex operational constraints, resulting static schedules often prove myopic and fail during real-world disruptions. We propose a simulation-optimization architecture that integrates traditional mathematical programming with a Markov Decision Process (MDP) framework to address these limitations.

The core innovation of this work is a hybrid approach that learns how to “tune” a MIP model rather than learning the schedule directly. A policy (π) maps the current system state - including, for example, the number of active collaborators and predicted demand peaks - to a set of policy parameters (θ_t). These parameters dynamically adjust the MIP’s requirements, allowing the model to remain deterministic while the policy selects the optimal level of robustness for each period.

The system is validated using a specialized simulator and real-world data from to model various sources of uncertainty. Rewards are calculated based on performance metrics such as service levels, missing hours, and the operational impact of schedule alterations. By advancing from deterministic baselines to a variable MDP-based policy, this research effectively bridges the gap between static optimization and dynamic real-world execution.

Keywords: *Workforce Scheduling, Markov Decision Process, Mixed-Integer Programming, Reinforcement Learning, Real-world*

Acknowledgments: Funded by the European Union’s Horizon Europe Programme under Grant Agreement No. 1011240406 and and COMPETE 2030 with operation code COMPETE2030-FEDER-01434100.

Learning Interpretable Dispatching Rules with AI for a Large-Scale Maintenance Provider

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Abstract

Complex assets (such as jet engines, avionics components, and medical imaging systems) are high-value equipment composed of numerous interconnected subcomponents that require specialized technical expertise and strict regulatory compliance to maintain.

Scheduling maintenance tasks for those assets is challenging due to both problem scale and multiple sources of uncertainty, such as dynamic arrivals, stochastic task durations, and unplanned work. These characteristics require continuous, real-time scheduling decisions.

However, previous approaches are based, in part, on exact and metaheuristic methods that generate full schedules, and thus exhibit limited scalability due to the computational burden they entail. Dynamic programming approaches, on the other hand, generate opaque policies. Dispatching rule-based approaches offer transparency and speed, but rely on manually generated rules with limited performance, or on complex AI-based rules developed primarily for synthetic datasets.

This study proposes learning interpretable dispatching rules for a large-scale real-world maintenance problem using genetic programming. Unlike previous studies, this paper relies on a set of realistic instances that closely reflect maintenance environments in terms of scale and uncertainty.

Computational experiments show that the evolved rules outperform benchmarks by around 4% in a wide range of realistic scenarios while maintaining a compact form. These improvements are estimated to yield significant monetary gains. The new rules cope well with extreme scenarios, demonstrating their generalisation and assuring they can be safely deployed in real environments.

Keywords: *Dynamic Scheduling, Genetic Programming, Dispatching Rules, Asset Maintenance*

Acknowledgments: Funded by the European Union’s Horizon Europe programme under Grant Agreement No. 101120406 (Project PEER).

Deterministic Scheduling of Time-Based Preventive Maintenance under Calendar and Usage Intervals

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Abstract

Most preventive maintenance optimisation models in the literature are condition- or reliability-driven. Yet, many networked critical infrastructure systems—operated by utilities and service providers—still rely on mandatory time-based preventive maintenance scheduling (TBPM), defined by strict calendar and/or usage intervals. This study addresses TBPM as a deterministic scheduling problem over a cyclical planning horizon, where maintenance tasks must be executed within prescribed maximum inter-visit times while coordinating workforce assignments across geographically distributed assets. We formulate an optimisation model that integrates calendar-time and equipment-usage criteria and minimises total maintenance and transportation costs while satisfying compliance constraints. The approach further exploits operational synergies by grouping tasks spatially and temporally, improving routing efficiency and resource utilisation. The model is applied to a real-world network of pumping stations operated by a Portuguese water utility. Results show that the proposed TBPM scheduling framework supports system-wide maintenance planning, guarantees compliance with mandatory intervals, and improves the efficiency of workforce deployment in networked infrastructure systems.

Keywords: *Maintenance scheduling, Workforce allocation, Critical infrastructure, Water utilities*

Acknowledgments: This work is co-financed by Component 5 - Capitalization and Business Innovation, integrated in the Resilience Dimension of the Recovery and Resilience Plan within the scope of the Recovery and Resilience Mechanism (MRR) of the European Union (EU), framed in the Next Generation EU, for the period 2021 - 2026, within project SMARTgNOSTICS, with reference 5. This work was also supported by the Portuguese Foundation for Science and Technology - FCT through PhD research grant under reference 2025.04109.BD.

Integrated Production and Transportation Scheduling with Multi-Load AGVs

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Abstract

Production scheduling is a critical in manufacturing systems where multiple jobs must be processed on different machines. Recent studies have emphasized the importance of integrating transportation resources into job shop scheduling problems to improve efficiency and reduce operational costs. In such systems, jobs are picked up at the load/unload area, delivered to the machine processing their first operation, and then transported between machines until all operations are completed. Until recently, much of the literature ignored transportation, treated it in a simplified manner, for example, by assuming unlimited AGV availability, or addressed production and transport sequentially; however, their simultaneous consideration is necessary to capture their strong interdependence, despite significantly increasing problem complexity.

In this work, we extend the job shop scheduling problem with transportation by considering Automated Guided Vehicles (AGVs) with multi-load capacity. Unlike most previous studies, which assume single-load vehicles, we investigate a more realistic setting in which the AGV fleet is limited and each vehicle can transport multiple jobs simultaneously. We develop a Biased Random-Key Genetic Algorithm (BRKGA) for this problem and evaluate it on benchmark instances.

The computational results show that multi-load AGVs can significantly improve system performance by reducing makespan, decreasing empty trips, and improving synchronization between transportation and production. We further show that a smaller fleet of higher-capacity AGVs can outperform a larger fleet of single-load vehicles, while also reducing space requirements and operational costs. These findings provide both methodological and managerial insight for the design of more efficient manufacturing systems.

Keywords: *Job shop scheduling, Integrated production and transportation scheduling, Multi-load Automated guided vehicles, Biased random-key genetic algorithm.*

Acknowledgments: Work co-funded by COMPETE 2030, Portugal 2030 and European Union (COMPETE2030-FEDER-01474600).

MC4: Geometry and Regularization in Nonconvex Optimization

Chair: Orizon Ferreira

Room: Santander (Floor 3)

Organizer: Orizon Ferreira

Why Study Spherical Convexity of Non-Homogeneous Quadratics and What Makes It Surprising?

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Abstract

This talk presents necessary, sufficient, and equivalent conditions for the spherical convexity of non-homogeneous quadratic functions. By examining criteria for determining spherical convexity, we identified unique properties that differentiate spherically convex quadratic functions from their geodesically convex counterparts in both hyperbolic and Euclidean spaces.

Since spherically convex functions over the entire sphere are constant, our analysis focuses on proper spherically convex subsets of the sphere. Our primary results concern non-homogeneous quadratic functions on the spherically convex set of unit vectors with positive coordinates. We also present extensions of our findings to more general spherically convex sets.

Additionally, the talk explores special cases of non-homogeneous quadratic functions where the defining matrix is of a specific type, such as positive, diagonal, or a Z-matrix.

This study not only provides useful criteria for spherical convexity but also reveals surprising characteristics of spherically convex quadratic functions, contributing to a deeper understanding of convexity in spherical geometries.

Keywords: *Spherical Convexity, Non-Homogeneous Quadratics, Z-matrix*

Subsampled cubic regularization method with distinct sample sizes for function, gradient, and Hessian

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Abstract

In this talk, we develop and study a subsampled cubic regularization method for finite-sum composite optimization problems, in which the function, gradient, and Hessian are estimated using possibly different sample sizes. By allowing each quantity to have its own sampling strategy, the proposed method offers greater flexibility to control the accuracy of the model components and to better balance computational effort and estimation quality. Such flexibility is particularly valuable in large-scale settings where the relative cost of evaluating these quantities can vary significantly. We establish iteration-complexity bounds for computing approximate first-order critical points and prove global convergence properties. In addition, we present numerical experiments that illustrate the practical performance of the proposed method.

Keywords: *Subsampled, cubic regularization method, gradient, Hessian*

Acknowledgments: CNPQ, FAPEG, FUNAPE-UFG.

A Cubic Regularization Method for Multiobjective Optimization

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Abstract

In this talk, we present a cubic regularization method for solving nonconvex multiobjective optimization problems. The method minimizes a regularized model of each objective, allowing approximate first- and second-order derivatives that adapt dynamically with the regularization parameter through a nonmonotone line search. We also discuss finite-difference implementations that make the method practical when derivatives are unavailable. Under mild assumptions, the algorithm achieves a worst-case complexity of order $O(\varepsilon^{-3/2})$ to compute an ε -approximate Pareto critical point and exhibits superlinear or even quadratic convergence when exact derivative information is available.

Keywords: *Cubic Regularization Method, Multiobjective Optimization, nonmonotone line search*

Acknowledgments: CNPQ, FAPEG, FUNAPE-UFG.

A subdifferential characterization via Busemann functions and applications to DC optimization on Hadamard manifolds

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Abstract

This presents properties of Busemann functions on Hadamard manifolds and their use in optimization algorithms in Riemannian settings. We present a new Busemann-based characterization of the subdifferential, which is particularly well suited to Riemannian optimization. In the classical Hadamard manifold framework, a subgradient provides a global lower model of a convex function expressed through the inverse exponential map. However, this model may fail to exhibit a useful convexity or concavity structure. By contrast, our characterization yields a concave bounding function by exploiting key properties of Busemann functions. We use this concavity to design and analyze difference-of-convex (DC) optimization methods on Hadamard manifolds. In particular, we reformulate the classical DC algorithm (DCA) for Riemannian contexts and study its convergence properties. We also report preliminary numerical experiments comparing the proposed Busemann DCA, which leads to geodesically convex subproblems, with the classical Riemannian DCA.

Keywords: *Busemann functions, Hadamard manifolds, DC algorithm*

Acknowledgments: CNPQ, FAPEG, FUNAPE-UFG.

Artillery Firing Shift with Two Registration Targets

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Abstract

Firing Shift is the shifting of artillery fire from one target ('registration') to another by the application of corrections determined from the adjustment of the initial firing data for the first target to the second ('application'). In this work a shift method based on simultaneous firing at two different registration targets rather than a single one is introduced. A method is presented for using the observed data to explicitly solve for four major environmental parameters which affect ballistic trajectories and subsequently update them for the application mission. The distribution of the remaining errors is determined and compared with that of other correcting algorithms. Numerical examples show the potential of the proposed method.

Keywords: *Military OR, Fire Transfer*

Acknowledgments: Use only when you need to acknowledge your research.

Optimizing Power Hubs and Subsea Connections in Pre-Salt Offshore Fields

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Abstract

The Brazilian pre-salt offshore fields present significant operational challenges due to the high energy demand for processing and injection operations. In this context, the implementation of power hubs emerges as a promising solution, providing dedicated electricity generation to relieve the load on consumer units and enable efficient offshore energy distribution. This research proposes an integrated approach for optimizing the network and subsea layout, considering the complexity of direct connections between hubs and production units, with central distributors or manifolds strategically employed to reduce this complexity. The objective is to identify configurations that maximize energy generation and transmission, meet the demand of consumer units, and comply with the physical and operational constraints of the subsea environment. Furthermore, the model accounts for the inherent uncertainty of offshore operations, particularly the fluctuations in energy demand from consumer units. Therefore, it aims to provide a decision-support framework for the planning and implementation of the offshore network that is safe, efficient, and economically viable, ensuring flexibility and robustness under uncertain conditions. The proposed integration is expected to contribute to systemic efficiency gains by reducing the load on production units, optimizing energy transmission, and simplifying the subsea layout.

Keywords: *Strategic planning, Optimization, Deep-water activities*

Acknowledgments: We gratefully acknowledge the support of the Offshore Technology Innovation Centre (OTIC), hosted by the University of São Paulo (USP) and sponsored by FAPESP – São Paulo Research Foundation (2022/03698-8). This work was also partially supported by the National Council for Scientific and Technological Development – CNPq (308792/2022-0).

Automated Design Framework for High-Efficiency Airfoils Using CFD-Synchronized Nelder–Mead Optimization

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Abstract

Modified trailing-edge (TE) geometries in wings and aerodynamic blades are a well-established passive strategy for noise control. However, such modifications often degrade aerodynamic performance, creating a trade-off between aeroacoustic and aerodynamic objectives. This study presents an automated airfoil design framework based on simulation-coupled Nelder–Mead optimization, in which modified TE designs are iteratively refined to maximize the lift-to-drag ratio (Cl/Cd) while satisfying a minimum lift constraint. The platform integrates computational fluid dynamics (CFD) directly within the optimization loop, enabling automated geometry updates, multi-step simulation execution, and adaptive simplex evolution. Applied to a standard NACA 0012 airfoil, the framework optimizes three design variables associated with TE modification. Two variables are bounded, while the third depends on one of the others. Evaluated under six flight conditions, the system demonstrates stable convergence in fewer than 25 iterations per condition, yielding Cl/Cd improvements up to 10% over a traditional serrated TE. Robustness and computational efficiency are enhanced through synchronized execution, zero-cost filtering of repeated or infeasible points, balanced case distribution, and adaptive simplex stabilization. The results show that the framework provides a reliable, computationally efficient solution for CFD-driven optimization problems involving competing aerodynamic and aeroacoustic objectives.

Keywords: *Trailing-edge, Wing Design, Optimization, Lift-to-Drag ratio*

A Metaheuristic Approach for Joint Layout and Power Smoothing in Airborne Wind Energy Farms

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Abstract

Airborne Wind Energy (AWE) is an emerging technology that harnesses stronger and more persistent winds at high altitudes using tethered flying devices. AWE farms are arrays of multiple AWE units operating within the same site, whose interactions and operating cycles must be coordinated to ensure efficient and stable power generation.

AWE farms face significant power fluctuations due to the periodic productive cycles of ground-generation units [1]. This work addresses the optimization of farm layout and power smoothing to ensure grid stability while maximizing energy production [1]. The problem is formulated as a MINLP model and, given its complexity, is solved by a Biased Random-Key Genetic Algorithm [2]. We discuss a "inside reel-in" retraction method that does not require complex synchronization among kites, allowing units to operate independently and significantly reducing output signal variations. Numerical experiments demonstrate that this strategy preserves near-maximal average power while significantly improving smoothness.

Keywords: *Airborne wind energy, Wind farms, Layout optimization, Synchronization, Power smoothing*

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Acknowledgments: This work is financially supported by FCT/MCTES (PIDDAC), through FCT Scholarship (Ref: 2025.03759.BDANA) and project 2022.02801.PTDC-UPWIND-ATOL (<https://doi.org/10.54499/2022.02801.PTDC>).

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Università degli Studi di Brescia, Italy

Title: Exact approaches for colorful components problems based on representative formulations**Chair:** Luís Gouveia**Room:** CGD (Floor 2)**Abstract**

In a node-colored graph, a colorful component is a connected subgraph in which no two nodes share the same color. Colorful components problems are graph modification problems that aim to transform a node-colored graph into a collection of colorful components by deleting edges. We consider three variants of colorful components problems that have been studied in the literature: the MOP, MCC, and MEC problems. For them, we propose new integer programming formulations based on representative nodes, where each component is identified by a representative, eliminating the symmetry inherent in classical assignment-based models. Moreover, we introduce a novel flow-based modeling approach to enforce component connectivity through additional variables and constraints. To further improve computational efficiency, we also develop a preprocessing procedure that reduces the number of variables in the representative formulations. Computational experiments demonstrate that the proposed models, particularly those using flow-based connectivity constraints, significantly outperform the current state-of-the-art approach, solving all benchmark instances to optimality within negligible computational time

TA1: Routing II**Chair:** Ana Paias**Room:** Amphit. 1 (Floor 4)

An iterated local search algorithm for the traveling purchaser problem

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Abstract

The Traveling Purchaser Problem (TPP) is a generalization of the Traveling Salesman Problem (TSP) in which a list of items must be acquired by visiting a subset of markets. The objective is to minimize the total cost sustained along the route, including purchasing and traveling costs. Due to the NP-hard nature of the problem, solving the TPP in an exact manner is computationally challenging, implying the need for heuristic approaches to obtain quality solutions efficiently. This study proposes an algorithm based on the metaheuristic Iterated Local Search (ILS), complemented by a route configuration procedure that adjusts the subset of markets in the solution. The ILS is tested in benchmark instances, providing a performance comparison with other methods. The computational experiment for the asymmetric instances reveals the effectiveness and efficiency of the ILS, outperforming previously published results with statistical significance. Additional experiments are presented for the symmetric instances, pointing to the competitiveness and versatility of the ILS in relation to other heuristic approaches used in the literature.

Keywords: *Traveling purchaser problem, Metaheuristics, Iterated local search, Route configuration.*

Acknowledgments: Portuguese National Funding supported this research from FCT - Fundação para a Ciência e a Tecnologia, under projects CEMAPRE / REM - UIDB/05069/2020 and 2024.04679.BD.

Optimal Policies for the Collection and Transportation of Product Returns

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Abstract

Product returns are a vital part of conducting business today, due to the emergence of e-commerce and the proliferation of generous return policies. Such policies, however, can backfire due to their costly operations and unpredictability. To address this impact, we develop and compare different collection policies to minimize the combined inventory and transportation costs in a three-echelon network consisting of (a) customers returning products to initial collection points (ICPs), (b) the ICPs, and (c) the centralized return centers (CRCs), which receive the products shipped from the ICPs after they are aggregated into larger shipments to leverage economies of scale. Collection policies are developed for the case of multiple products with a single ICP and multiple CRCs, using (1) individual shipment policy where ICPs ship out products to individual CRCs and (2) a combined shipment policy that allows shipping and routing from an ICP to multiple CRCs. Mathematical models are developed to determine the collection period at the ICPs to minimize the combined inventory and transportation costs for these policies. An efficient algorithm is developed, and results are presented with an experimental dataset for the case of a single ICP and three CRCs, each of which processes 4 different products.

Keywords: *Product Returns, Optimization, Reverse Logistics*

A matheuristic and a hybrid algorithm for the asymmetric period travelling salesman problem

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Abstract

Considering a time horizon with different periods, the Period Travelling Salesman Problem (PTSP) is defined on a directed graph with a depot and a set of nodes, in which each node has a required number of periods to be visited by the salesman. The objective of the PTSP is to assign each client to as many periods as it is required while determining the set of circuits, one for each period, with minimum cost. Each circuit starts and ends at the depot and visits the nodes assigned to that specific period only once.

The PTSP combines an assignment and a routing problem in a single problem, making it very difficult to solve, even for instances with few nodes. In this work we study empirical characteristics of a non-valid formulation for the PTSP that is period-aggregated and propose a matheuristic based on this formulation. Additionally, we develop an Iterated Local Search algorithm and introduce a hybrid heuristic that combines elements from both heuristic approaches. We assess the computational performance of the three heuristics using both benchmark instances from the literature and a newly generated set of asymmetric instances, with up to 443 nodes and 5 periods. All proposed methods produced high-quality solutions. For large-scale instances, the matheuristic enhanced with a simple local search procedure was able to find the global optimum in many cases. In particular, it solved an asymmetric instance with 443 nodes and 5 periods to optimality within 10 minutes (see [1]).

Keywords: *Combinatorial optimization, Heuristics, Mixed integer linear programming*

References

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General formulation framework for combined subtour and path elimination constraints in multi-depot routing problems

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Abstract

In this work, we study multi-depot routing problems, which involve a set of clients to be visited from a set of two or more depots. These problems require simultaneously subtour elimination constraints, i.e., constraints to prevent circuits without a depot, and path elimination constraints, i.e., constraints to prevent circuits with more than one depot. We propose a general framework to derive compact formulations in which these two families of constraints are handled jointly, based on the idea of mixing the modeling of positions or flows to handle subtours and currents to handle infeasible paths between depots. We present theoretical comparisons of these models, as well as computational results on linear programming relaxation values and time to optimality in benchmark instances.

Keywords: *Multi-depot routing, Subtour elimination, Path elimination, Compact formulations*

Acknowledgments: This work is financed by Portuguese funds through the FCT - Foundation for Science and Technology. I.P.. under projects UID/06522/2025 (ISEG Research) and UID/97/2025 (CEGIST).

TA2: Environment, Agriculture and Forestry

Chair: Miguel Constantino

Room: Amphit. 2 (Floor 4)

Organizer: Marta Mesquita & Miguel Constantino

Optimization of harvest scheduling under clearcut area and clearcut proximity constraints

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Abstract

This study addresses a forest harvesting scheduling optimization problem subject to two types of spatial restrictions: a maximum area threshold for each clearcut and a requirement for minimum-width buffers of standing forest surrounding each clearcut. The objective is to maximize the net present value of harvesting revenues, incorporating penalties for the formation of clearcuts. Both the clearcut constraints and the associated penalties are designed to mitigate the environmental impact of clearcutting. We propose an extension of the “bucket model” that includes additional constraints on bucket connectivity and two distinct definitions of stand adjacency (“connected-bucket model”). This model is solved using a Dantzig-Wolfe decomposition via branch-and-price, and the results are compared against the connected-bucket model solved directly by CPLEX. Computational experiments on real-world instances demonstrate that the branch-and-price approach outperforms the alternative in most cases.

Keywords: *Integer programming, Dantzig-Wolfe decomposition, Branch-and-price, Bucket formulation*

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Designing Resilient Forest Road Networks for Wildfire Response

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Abstract

Wildfires pose an increasing threat to ecosystems, infrastructure, and human life, particularly in fire prone regions such as northern Portugal. Effective forest management and emergency response depend critically on the design of forest road networks that ensure rapid access to all areas while providing safe evacuation alternatives when primary routes become compromised. This work proposes a mathematical optimization framework for designing resilient forest road networks that jointly address accessibility, safety, and cost efficiency. We consider a forested region containing a set of sites that must be reachable. The area includes an existing road network and a set of potential forest roads that may be constructed. To determine which new roads should be added to complement the municipal network, we formulate a two unit network flow-based MILP model. The model ensures that each site lies within a predefined maximum distance from a road and is connected to the existing network through at least two node disjoint paths. To promote fast and safe access, we further impose that the combined length of these two paths does not exceed a specified threshold. The objective is to minimize construction costs while satisfying accessibility and dual path connectivity requirements under enhanced safety conditions. Applied to a real case in northern Portugal, the model generated cost-effective road layouts that significantly improved operational access and evacuation safety.

Keywords: *Forest Road Network, Wildfire Response, Mixed-Integer Linear Programming, Network flow-based Model, Dual Access Node-disjoint Paths*

Dependence and moral hazard constraints in optimal reinsurance design

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Abstract

The optimal reinsurance problem for multiple dependent risks is examined from the direct insurer's viewpoint. The objective is to determine the treaty that maximizes expected utility, assuming separate negotiation of reinsurance for each risk and allowing for different moment-based premium principles. The dependence structure among risks is kept fully general.

The analysis is conducted both with and without additional constraints, such as the Lipschitz condition commonly imposed to mitigate moral hazard. Building on the optimality conditions established in M. Guerra and A. B. Moura (2021), the effect of the Lipschitz constraint on the optimal treaty is investigated. Numerical methods are used to compute optimal solutions for general dependence structures, with and without constraints, and numerical examples are provided to compare the resulting treaties.

Keywords: *Risk Management, Reinsurance, Expected Utility, Dependence, Moral-Hazard*

Acknowledgments: The authors were partially funded by FCT – Fundação para a Ciência e Tecnologia (Portugal), with national funding through research grants UID/6522/2025 and EXPL/EGE-ECO/0886/2021.

The l_1 exact E -penalty function method for constrained nonconvex optimization problems

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Abstract

In this study, we introduce and apply the l_1 exact E -penalty function method to a novel class of generalized convex optimization problems. We focus on the E -differentiable constrained optimization problem, which encompasses both inequality and equality constraints involving E -invex functions. Our findings demonstrate the existence of a finite threshold for the penalty parameter, equal to the maximum absolute value of the Lagrange multiplier. When the penalty parameter exceeds the threshold, a E -optimal solution with E -invex functions is equivalent to a minimizer of E -penalized.

Keywords: l_1 exact penalty function method, E -differentiable function, E -invex function, optimality conditions

AMS Classification: 90C26; 49M30; 90C30.

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Symmetric gauge theory for cutting-plane methods in semidefinite programming

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Abstract

We present a general cutting-plane framework for semidefinite programming based on symmetric gauge theory. By normalizing separators via gauges, we formulate a separation problem and derive a closed-form oracle. The resulting cuts unify several classical approaches while generating infinitely many new families with tunable structure. We embed this oracle within a Kelley-type algorithm initialized from low-cost relaxations and prove convergence to the semidefinite optimum. Numerical experiments show that the proposed cuts are computationally efficient and competitive with existing methods, offering a trade-off between strength and tractability. Some target applications include quadratic optimization with box constraints (BoxQP) and sparse principal component analysis (sparse PCA).

Keywords: *semidefinite programming, cutting-plane method, symmetric gauges, sparse principal component analysis, non-convex quadratic optimization*

On polling directions for randomized direct-search approaches

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Abstract

The use of random strategies to address a problem in a deterministic setup is becoming more common in optimization after its success in machine learning and artificial intelligence. In some domains the advantages of using random strategies are known but the question here is whether, for a deterministic setup of the optimization problem at hand, randomized methods are true competitors with deterministic ones. Introducing randomization into otherwise deterministic direct-search approaches has been shown to lead to excellent computational performance, particularly when considering a reduced number (as low as two) of random poll directions at each iteration. In this study several randomized direct-search strategies are tested considering different sets of polling directions. The performance of the approaches considering different sets (and different number) of poll directions are assessed using the CUTer test problems suite. Our computational tests made it quite clear that greater distance and a greater number of directions were a competitive advantage. Therefore, we proposed random bases, but with directions as far apart as possible - this was the reasoning behind proposing random, orthogonal polling directions. The results indicate that, in general, reducing the number of directions leads to a decrease in the quality of the results, and considering orthogonal random directions has proven to be a competitive approach.

Keywords: *direct-search, polling directions, randomization*

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An Integrated Agent-Based and Optimisation Approach for Evaluating Lock Performance Under Convoy Splitting: The Case of the Bariri Lock

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Abstract

Inland waterway freight transport is characterized by the movement of vessels along interconnected networks of rivers, canals, and lock systems. In many inland navigation corridors, freight operations are predominantly performed by river convoys composed of a towboat and a set of barges. On waterways equipped with locks, the dimensions of the lock chamber impose physical constraints on the maximum convoy size that can be transposed in a single lockage cycle. Over time, in response to the need to preserve economic competitiveness, river convoys have increased the number of barges and, consequently, their transport capacity. However, this operational evolution has also introduced additional complexity to lock transit, since larger convoys frequently require double lockage procedures, thereby increasing transit time and contributing to the formation of queues at critical bottlenecks. The main objective of this research is to evaluate the effects of convoy splitting operations and mooring area configurations on the operational efficiency of the Bariri Lock, located on the Tietê-Paraná Waterway in southeastern Brazil. To this end, an agent-based simulation model was developed in which operational decisions rely on an optimisation model and heuristic rules to determine the sequencing of lock transits, with the objective of minimising waiting times. The results provide a quantitative basis for assessing the operational implications of alternative engineering and operational configurations.

Keywords: *Agent-Based Modelling, Optimisation, Heuristic, Inland Navigation, Locks*

Acknowledgments: This research is funded by FAPESP - The São Paulo Research Foundation, Grant 2025/07367-4.

Truck Scheduling for Congestion Mitigation in Container Terminals

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Abstract

Maritime trade has been continuously growing in recent decades, increasing pressure on container terminals and intensifying the need for more effective management systems. A major source of inefficiency in landside operations stems from rising truck arrivals, whose stochastic nature leads to unbalanced workload distribution and inconsistent resource utilization, worsening congestion and overall operational efficiency. This work addresses the landside congestion problem by improving coordination between truck scheduling and internal operational constraints. A constructive heuristic is proposed, integrating time-window allocation with yard relocation considerations and multi-criteria truck scheduling into a unified decision-support framework. The approach explicitly accounts for crane capacity limits and container stack conditions, while balancing carrier appointment preferences with operational impact. Preliminary computational experiments, based on a large Portuguese container terminal, show improved workload alignment, increasing the number of trucks handled within effective capacity limits while maintaining high service responsiveness. The framework generates operationally meaningful schedules suitable for practical implementation, supporting more stable and congestion-resilient terminal operations.

Keywords: *Port logistics, Truck scheduling, ; Decision support systems, Optimization.*

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Multi-Objective Optimization of Container Transportation in Multi-Depot Logistics Networks

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Abstract

As the complexity of global supply chains grows, maritime transport, especially container terminals, faces increasing pressure to better integrate sea-side operations with land-side distribution. In this context, planning and dispatching container trucks in hinterland transport is a key challenge, made harder by networks spanning multiple terminals and depots with vehicles and containers dispersed across yards and inland facilities. This work aims to tackle this problem by designing a decision support system based on a Multi-Objective Mixed-Integer Linear Programming to reduce total transportation time and cost. The model determines the best vehicle assignment and task scheduling, accounting for mixed vehicle fleets, driver working time regulations, multiple depots and terminals, time windows for pickups and deliveries, and managing both full and empty containers. Preliminary results show practically interesting trade-offs between these conflicting objectives. Minimizing total transportation time tends to favour the use of conventional fuel-powered trucks, given their greater operational flexibility when compared to electric vehicles. Conversely, when cost and environmental considerations are given more weight, the model increasingly allocates electric vehicles, despite their limitations in terms of range and charging requirements.

Keywords: *Global supply chains, Logistics networks, Container terminals, Truck scheduling, Multi-objective optimization*

Acknowledgments: This work is funded by national funds through FCT-Fundação para a Ciência e a tecnologia IP, under the support UID/50014/2025, and co-financed by Component 5 – Capitalization and Business Innovation, integrated in the Resilience Dimension of the Recovery and Resilience Plan, within the scope of the Recovery and Resilience Mechanism (MRR) of the European Union (EU), framed in the Next Generation EU, for the period 2021 – 2026, within project NEXUS, with reference 53.

Integrated Scheduling of AGVs and Yard Cranes Considering Charging Decisions

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Abstract

Container terminals are becoming increasingly automated, highlighting the need for efficient management of Automated Guided Vehicles (AGVs). At the same time, environmental concerns have encouraged terminals to replace diesel-powered vehicles with battery-powered AGVs (B-AGVs), making charging a critical component of operations. Charging decisions directly affect AGV's availability and synchronization with quay and yard cranes, thereby impacting overall system performance.

In this talk, we address the integrated scheduling of B-AGVs and cranes while explicitly accounting for battery charging decisions. We formulate the problem as a mixed-integer linear programming (MILP) model and derive tight lower bounds on its optimal value. To handle realistically sized instances, we develop a rolling horizon heuristic capable of producing near-optimal solutions within practical computational times.

Keywords: *Scheduling, AGVs, Charging decisions, rolling horizon*

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Acknowledgments: This work is financed by Portuguese funds through the FCT - Foundation for Science and Technology. I.P.. under the project UID/06522/2025 (ISEG Research).

TA5: Derivative-Free Optimization II

Chair: Everton Silva

Room: Santander (Floor 3)

Organizer: Everton Silva & Yiwen Chen

A Hybrid Direct Multisearch and Projected Simplex Gradient Method for Multiobjective Problems on Convex Sets

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Abstract

We propose a new Derivative-free Optimization (DFO) approach for solving Multiobjective Optimization Problems (MOP) on convex sets. The feasible set is assumed to be the nonempty intersection of a finite collection of closed convex sets. Our iterative approach alternates between steps that use Direct Multisearch (DMS) and a Spectral Projected Gradient method for MOP (SPG-MOP), replacing the real gradient by a simplex gradient, under a DFO approach. Convergence properties are established under standard assumptions usually associated with DFO methods. Some preliminary numerical experiments are reported to illustrate the performance of the proposed algorithm, particularly by comparing it with a classical DMS method. Our results indicate that the hybrid algorithm is a robust and effective approach for this class of problems.

Keywords: *Derivative-free Optimization, Convex Constrained Optimization, Simplex Gradient, Direct Multisearch*

Pareto sensitivity, most-changing sub-fronts, and knee solutions

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Abstract

When dealing with a multi-objective optimization problem, obtaining a comprehensive representation of the Pareto front can be computationally expensive. Furthermore, identifying the most representative Pareto solutions can be difficult and sometimes ambiguous. A popular selection are the so-called Pareto knee solutions, where a small improvement in any objective leads to a large deterioration in at least one other objective. In this paper, using Pareto sensitivity, we show how to compute Pareto knee solutions according to their verbal definition of least maximal change. We refer to the resulting approach as the sensitivity knee (snee) approach, and we apply it to unconstrained and constrained problems. Pareto sensitivity can also be used to compute the most-changing Pareto sub-fronts around a Pareto solution, where the points are distributed along directions of maximum change, which could be of interest in a decision-making process if one is willing to explore solutions around a current one. Our approach is still restricted to scalarized methods, in particular to the weighted-sum or epsilon-constrained methods, and require the computation or approximations of first- and second-order derivatives. We include numerical results from synthetic problems that illustrate the benefits of our approach. This is joint work with Tommaso Giovannelli and Marcos M. Raimundo.

Keywords: *knee solutions, most-changing sub-fronts, multi-objective optimization, Pareto sensitivity, multi-criteria optimization*

Worst-case complexity analysis of derivative-free methods for multi-objective optimization

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Abstract

In this work, we are concerned with the worst case complexity analysis of *a posteriori* methods for unconstrained multi-objective optimization problems where objective function values can only be obtained by querying a black-box. We present two main algorithms, namely **DFM0strong** and **DFM0light**, which are based on a linesearch expansion technique. In particular, **DFM0strong** requires a complete exploration of the points in the current set of non-dominated solutions, whereas **DFM0light** only requires the exploration around a single point in the set of non-dominated solutions. For these algorithms, we derive worst case iteration and evaluation complexity results. In particular, the complexity results for **DFM0light** align with those recently proved in [1] for a directional multisearch method. Furthermore, by exploiting an expansion technique of the step, we are also able to give further complexity results concerning the number of iterations with a measure of stationarity above a prefixed tolerance.

Keywords: *Multi-objective optimization, Derivative-free methods, Worst-case complexity bounds*

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A Direct Multisearch Approach (DMS) for Many-Objective Derivative-Free Optimization

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Abstract

From a theoretical point of view, Direct Multisearch (DMS) was developed for continuous constrained multiobjective derivative-free optimization, with a general number of components in the objective function. However, the algorithmic performance was never assessed in problems with more than three objectives. In this work, we propose DMS-Reduction, a variant of DMS based on reduction approaches, using correlation and sketching techniques. This approach is an attempt to tackle larger problems, in what respects the number of components of the objective function and the number of variables. At each iteration, the reduction in the number of components of the objective function to be optimized has the possible additional benefit of conducting to a reduction in the number of variables to be considered, since there could be the case that not all variables are related to the objective function components selected. We will detail the proposed algorithmic structure and report promising numerical results in addressing many-objective optimization problems.

Keywords: *Derivative-free Optimization, Black-box optimization, Many-Objective Optimization, Direct Multisearch*

Acknowledgments: The authors were funded by national funds through FCT - Fundação para a Ciência e a Tecnologia I.P., under the scope of the projects UID/00297/2025 and UID/PRR/00297/2025 (Center for Mathematics and Applications). The first author was also funded by national funds through FCT - Fundação para a Ciência e a Tecnologia I.P., under the scope of the project UI/BD/151246/2021.

Workforce Planning in the Portuguese Navy: A Linear Programming Approach

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Abstract

In military organizations, effective workforce planning requires balancing operational personnel requirements with equitable career development policies. In the Portuguese Armed Forces, an essential instrument for workforce planning is the Special Personnel Table (SPT), which defines the authorized allocation of positions by category, rank, and specialization. This paper addresses the design of a Reference SPT (RSPT) for the Portuguese Navy to guide the annual definition of actual SPTs. We propose a family of five linear programming models that optimize position allocation under legal limits, operational requirements, availability rates, and career objectives. The models address two career objectives: balanced progression across ranks and balanced time in ship-based versus shore-based assignments. Using goal programming, these objectives are formulated as hard or soft requirements, yielding five policy regimes from strict enforcement of both targets to fully flexible trade-offs. Implemented in MATLAB, the framework enables parameter configuration and scenario analysis under different policy assumptions. The framework was validated against current SPTs and refined through feedback from institutional decision-makers, confirming its potential to accommodate different managerial priorities within a unified quantitative decision-support tool.

Keywords: *linear programming, goal programming, scenario analysis, decision support systems, workforce planning*

Choice-based Availability Controls for Urban Carsharing Revenue Management

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Abstract

Urban carsharing services provide flexible, affordable and green mobility options where customers can pick up a car from one station and return it to the same station (round trip) or any other station in the car-sharing network (one-way). While one-way car-sharing offers a higher degree of mobility flexibility to customers, it may cause operational challenges such as fleet unbalance between multiple rental stations. Carsharing operators have employed various vehicle relocation strategies to ensure the availability of sufficient vehicles across rental stations. In this study, we adopt the user-based relocation strategy where customers are offered incentives (e.g., fare discounts) to drop off vehicles to alternative destinations. We estimate a Multinomial logit (MNL) choice model based on collated responses from a discrete choice experiment implemented in a survey. We build the choice model into a dynamic program to determine the optimal destination-fare discount combinations offered to each arriving customer, which maximises the total expected revenue over a finite time horizon. To address the computational complexity of the dynamic program, we propose two approximation approaches: a choice-based deterministic linear program and a decomposition method. An extensive numerical study shows the effectiveness and efficiency of the proposed approaches, especially the latter.

Keywords: *Carsharing, network revenue management, discrete choice modeling, dynamic programming, decomposition*

Endogenous identification of worst-practice frontiers in hospital efficiency: An inverted value efficiency approach

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Abstract

Efficiency analysis in healthcare has traditionally focused on benchmarking best-performing units. However, persistent inefficiency in public service systems often arises from structural worst-practice configurations rather than deviations from excellence.

This paper presents an optimisation-based framework for worst-practice benchmarking grounded in Inverted Value Efficiency Analysis (IVEA). A central methodological challenge in inverted frontier models lies in the specification of the Least Preferred Solution (LPS), which is commonly imposed exogenously. We address this limitation by reformulating LPS identification as an endogenous preference optimisation problem. Using population-based metaheuristics, namely Particle Swarm Optimisation and Genetic Algorithms, extreme undesirable value structures are inferred directly from observed production data.

The resulting LPS is embedded within an IVEA model to evaluate hospital inefficiency relative to a common worst-practice benchmark. In a post-benchmarking stage, machine learning techniques are applied to support classification, clustering, and monitoring, without altering the normative structure of the efficiency model.

An empirical application to Portuguese public hospitals illustrates how the proposed approach uncovers persistent worst-practice regimes that remain obscured under conventional best-practice analysis. The framework highlights the role of optimisation as a tool for endogenous preference discovery in efficiency modelling and offers a complementary perspective for performance diagnosis in complex public systems.

Keywords: *Inverted efficiency, Preference optimisation, Metaheuristics, Data envelopment analysis, Hospital benchmarking*

Selected methods for reducing inconsistency in pairwise comparison matrices

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Abstract

Pairwise comparison matrices are widely used in multicriteria decision making (MCDM). One of the main challenges of this technique is inconsistency, which frequently arises and may lead to significant difficulties in the decision-making process. Hence it is necessary to keep inconsistency below an acceptable threshold.

The literature offers several approaches for reducing the inconsistency of pairwise comparison matrices (PCMs). In practice, these techniques typically suggest modifications to the original judgments of the decision maker in order to achieve an acceptable inconsistency level. It is important to note that such changes should be as small as possible, while simultaneously aiming to minimize the level of inconsistency. In this talk, I present the main directions of optimization models developed for minimizing the inconsistency of PCMs, focusing either on one of these objectives or on combining both.

In addition to previously published, well-known studies, several methods developed by the author are also discussed and compared to each other. In my talk I also present the results of computational simulations conducted to analyze the computational efficiency of the proposed techniques.

Finally, beyond introducing new methods, the main difficulties and challenges associated with the development of effective inconsistency reduction techniques are highlighted.

Keywords: *Pairwise comparison, Inconsistency, Multi-Criteria Decision Making*

Tom Van Woensel

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Title: Two-Echelon Vehicle Routing Problems: A short review, some models and directions for future research

Chair: Filipe Alvelos

Room: CGD (Floor 2)

Abstract

In the two-echelon vehicle routing problem (2E-VRP), the distribution network is split into two echelons. Different vehicles are operated on the first and second echelon to maintain economies of scale and adhere to any vehicle restrictions that may be present in either echelon. Intermediate facilities are located at the borders of the echelons to facilitate the consolidation and transshipment of goods between echelons. Examples of two-echelon distribution systems include express delivery, grocery and hypermarket products distribution, multi-modal freight transportation, city logistics, and ecommerce and home delivery services. In recent years, the body of literature on the 2E-VRP has expanded significantly. Many research papers have appeared in the scientific literature so far, which underlines both the academic and practical relevance of 2E-VRPs. In this review, we structure and revise all literature on the 2E-VRP. Mathematical formulations and benchmark datasets used to test and to evaluate new algorithms are reviewed and discussed.

TB1: Arc Routing**Chair:** Raquel Bernardino**Room:** Amphit. 1 (Floor 4)**Patrolling the Amazon River Navigation Network**André Bergsten Mendes ¹, Yanko Veríssimo Costa ², Joyce Azevedo Caetano ³, Laura Bahiense ⁴¹*Department of Naval Architecture and Ocean Engineering, University of Sao Paulo, Brazil, andbergs@usp.br*²*Department of Naval Architecture and Ocean Engineering, University of Sao Paulo, Brazil, yanko@usp.br*³*Department of Naval Architecture and Ocean Engineering, University of Sao Paulo, Brazil, joycecaetano@usp.br*⁴*Systems Engineering and Computer Science Program, Alberto Luiz Coimbra Institute – Graduate School and Research in Engineering, Federal University of Rio de Janeiro, Brazil, laura@cos.ufrj.br***Abstract**

The Amazon River and its tributaries form an extensive network spanning over 10,000 km, comprising numerous significant waterways. These rivers are vital for regional connectivity, supporting trade and commerce, and providing essential access to services for local communities. Effective patrolling is critical for combating illegal activities, preserving biodiversity, and safeguarding local populations. This research focuses on developing patrolling routes for the Amazon River navigation system, conceptualized as a variation of the Periodic Arc Routing Problem (PARP).

The study incorporates factors such as: i. extended navigation time spans, with some routes lasting up to 30 days; ii. different visitation frequencies required for each river segment; iii. minimum and maximum intervals between visits to arcs; iv. fleet sizing decisions to achieve the desired visitation frequencies.

A Mixed Integer Programming (MIP) formulation is proposed and evaluated using real-world data. Heuristic solutions are also developed, based on a probabilistic construction heuristic followed by local search procedures.

Keywords: *Periodic arc routing, Patrolling, Fleet size, Amazon basin*

Acknowledgments: This research is sponsored by The National Council for Scientific and Technological Development - CNPq, Brazil, under grant # 407587/2022-5.

Branch-and-Cut and Kernel Search Methods for the Periodic Drone Arc Routing Problem with Irregular Services and Maximum Benefit

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⁴*Departament de Matemàtica Aplicada, Universitat Politècnica de València, Country, jmsanchis@mat.upv.es*

Abstract

This work introduces the Periodic drone Arc Routing Problem with Irregular Services and Maximum Benefit, a novel continuous optimization problem motivated by surveillance and monitoring operations performed by autonomous drones. A set of linear features must be periodically serviced over a multi-day planning horizon, where each feature is associated with several subsets of allowed service days and a minimum frequency requirement. Each service yields a benefit, and additional services beyond the minimum are permitted when flight autonomy allows, generating extra benefit. Unlike ground-vehicle routing models, drones can enter and leave a line at any point, making the underlying problem continuous. To obtain tractable solutions, required lines are approximated by dense polygonal chains, giving rise to a discrete counterpart: the Periodic Arc Routing Problem with Irregular Services and Maximum Benefit. We propose an integer programming formulation incorporating connectivity, parity, and max-length constraints, and develop a branch-and-cut algorithm strengthened with tailored separation routines. Additionally, a matheuristic based on Kernel Search is designed to handle larger instances efficiently. Computational experiments on benchmark networks have been carried out to assess the effectiveness of the proposed methods.

Keywords: *periodic routing, drone arc routing problems, branch and cut, kernel search*

Acknowledgments: This work was supported by grant PID2021-122344NB-I00 funded by MCIN/AEI, Spain/10.13039/501100011033 and “ERDF A way of making Europe”.

Planning the Inspection of Electric Lines with Drones

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Abstract

This study addresses the Location-Ground Aerial Routing Problem (LGARP). The problem aims to optimize drone routes for the inspection of electric lines in Portugal, managed by EDP Labelec, the partner company in this project.

Given a set of electric lines requiring inspection, the objective is to determine the optimal sequence in which they should be inspected by the aerial vehicles (drones). The drones are transported by a van between a set of candidate parking locations, from which they are launched to start and end their aerial routes. Therefore, it is necessary to jointly determine a routing plan for the van (node routing) and for the drones (arc routing), ensuring their synchronization through the use of the same parking locations. The location component of the problem consists of selecting which parking sites should be activated.

The problem is formulated as a mixed-integer linear programming model and solutions are obtained using CPLEX on smaller real-world instances generated from a Geographic Information System (GIS) provided by EDP Labelec. These solutions are evaluated based on total inspection time, as well as on feedback from the EDP Labelec team, to ensure their practicality for real-world field implementation.

Keywords: *Arc Routing, Mixed Integer Programming*

Acknowledgments: Research supported by the FCT (Fundação para a Ciência e a Tecnologia – Portuguese Foundation for Science and Technology) through project UID/06522/2025.

TB2: Optimization in Graphs

Chair: Daniela Santos

Room: Amphit. 2 (Floor 4)

Organizer: Daniela Santos

Solver-Independent Computational Modelling of the Clique-Partitioning Problem

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Abstract

The ROAR-NET API specification defines an interface for modelling optimisation problems that allows a wide range of randomised optimisation algorithms to be applied to such problems in a black-box manner. The API abstracts the exploration of combinatorial decision spaces through neighbourhood structures, where the relation between neighbouring solutions is established in terms of small changes called ‘moves’.

The clique partitioning problem is a combinatorial optimisation problem that consists in partitioning the set of vertices of a complete undirected weighted graph so as to maximise the total weight of the internal edges of the resulting subgraphs. Modelling this problem under the ROAR-NET API entails the problem-specific instantiation of abstract constructive-search and local-search mechanisms that are exposed to optimisation algorithms in a standardised way. The presentation will zoom in on the development of such a model and discuss the challenges and opportunities afforded by the approach.

Keywords: *Combinatorial optimisation, Problem modelling, ROAR-NET API, Graph partitioning, Randomised algorithms*

Acknowledgments: This presentation is based upon work from COST Action Randomised Optimisation Algorithms Research Network (ROAR-NET), CA22137, supported by COST (European Cooperation in Science and Technology). Carlos M. Fonseca’s work is funded by national funds through FCT – Foundation for Science and Technology, I.P., within the scope of the research unit UID/00326 – Centre for Informatics and Systems of the University of Coimbra. Andreia P. Guerreiro acknowledges the support by Portuguese national funds through FCT, under projects 2022.08367.CEECIND/CP1717/ CT0001, UID/50021/2025, UID/PRR/50021/-2025, 2023.14280.PEX.

Cardinality-Based Extended Formulations and Cutset Inequalities for the Steiner Tree Problem

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Abstract

The Steiner Tree Problem (STP) is a classical NP-hard combinatorial optimisation problem with broad applications in network design, including VLSI circuit layout, optical and wireless communications, and supply chain logistics. Given a graph with a designated root and a subset of terminal nodes, the objective is to find a minimum-cost arborescence spanning all terminals. Achieving tight LP relaxations remains central to practical exact solution via branch-and-bound.

We introduce two cardinality-based extended Mixed-Integer Programming formulations, EMCF-1 and EMCF-2, extending the standard multicommodity flow (MCF) formulation. Arc cardinality—the number of commodities routed through an arc—is used to disaggregate both design and flow variables, exploiting the discrete combinatorial structure of feasible flows. We prove equivalence with MCF and show that disaggregation systematically tightens the LP relaxation.

Three families of cardinality-based cutset valid inequalities are introduced: the k-arc cardinality cutset inequality, extended cut, and the cardinality matching inequality. These cuts bound commodity flow across network cutsets and are incorporated via a cutting-plane procedure. Computational experiments on SteinLib I080/I160 instances (Gurobi 10.0, 2-hour limit) show EMCF-2 with cuts outperforms MCF and path-hierarchy formulations [1] on several instances, with stronger root bounds and faster gap closure despite larger size.

Keywords: *Steiner tree problem, cardinality formulation, cutset inequalities, network design, branch-and-cut*

References

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Ensuring Connectedness in the Multiobjective Quasi-Clique Problem

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Abstract

Given a graph $G = (V, E)$, the Multiobjective Quasi-clique (MOQC) problem consists of finding a subgraph G_S , induced by $S \subseteq V$, that maximises both edge density and vertex cardinality. This problem arises in applications such as social, biological, and telecommunication networks, where identifying cohesive and sufficiently large groups is essential. Recent exact approaches for the MOQC problem rely on scalarisation techniques combined with local search. However, they do not explicitly ensure connectedness, which may lead to subgraphs with isolated components that are not meaningful in many practical contexts, such as community detection. In this work, we extend these approaches by incorporating connectedness constraints, restricting the search to connected subgraphs. Although ensuring connectedness significantly enhances the applicability of the methods to practical domains requiring this property, computational experiments on benchmark instances show that this comes at the expense of increased computational effort, leading to a notable impact on running times. These results highlight an important trade-off between the practical relevance of the solutions and computational efficiency when solving the MOQC problem.

Keywords: *Quasi-clique, Graph Optimisation, Connectedness Property, Multiobjective Optimisation*

Acknowledgments: Daniela Scherer dos Santos acknowledges the Foundation for Science and Technology (FCT) for the Ph.D. fellowship 2022.12082.BD. This work is funded by national funds through FCT – Foundation for Science and Technology, I.P., within the scope of the research unit UID/00326 - Centre for Informatics and Systems of the University of Coimbra. Pedro Martins is also supported by national funds through FCT, under the project UID/04561/2025. This work is based upon work from COST Action Randomised Optimisation Algorithms Research Network (ROAR-NET), CA22137, supported by COST (European Cooperation in Science and Technology)

A Classification-based Heuristic Approach for The Covering Tour Problem with path Upgrades

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Abstract

This work addresses the Covering Tour Problem with Path Upgrades (*CTP-PU*), a routing problem where a tour must be designed to cover all nodes in a given graph, while selected arcs can be upgraded within a budget to extend coverage through shortest paths. The problem generalizes classical covering-tour variants and introduces operational decisions on path-based upgrades, making it computationally challenging. To solve the *CTP-PU*, we propose a novel heuristic approach that integrates data science and machine learning techniques with combinatorial optimization methods. First, node-level features are extracted to describe structural and operational properties. Then, a classifier predicts whether each node should belong to the tour or be covered. These predictions are used to fix or relax variables in a reduced MILP model, which is solved iteratively to guarantee feasibility and improve the solution. Random Forest and Naive Bayes classifiers are trained on optimal solutions of small instances and tested on medium- and large-scale cases. Computational experiments on three different sets of instances with up to 101 nodes show that the proposed method achieves solution quality very close to the model, while drastically reducing the computation running time. The approach proves robust across different instance types and parameter settings, confirming its ability to generalize to new and larger instances.

Keywords: *Routing, Covering, Machine Learning, Heuristic, Classification*

References

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Cyclic Exchange Heuristics for the Hamiltonian p -Median Problem

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Abstract

Given a positive integer p and a weighted undirected graph $G=(V,E)$, we study a problem in which the objective is to find a minimum weight set of p elementary cycles partitioning the vertices of G known as the Hamiltonian P -Median Problem (HpMP). We study the effectiveness of cyclic exchange heuristics as a neighbourhood exploration for the HpMP. We also compare it to classical neighbourhood-based heuristics to assess the contribution of cyclic exchanges.

Keywords: *Combinatorial Optimization, Heuristics, Mixed Integer Programming*

References

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New valid inequalities and projection results for the Hamiltonian p -Median problem

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Abstract

Given an edge-weighted undirected graph $G = (V, E)$ and an integer p , we consider the Hamiltonian p -Median problem (HpMP), in which one seeks the minimum total cost set of exactly p node-disjoint cycles spanning all nodes of the graph. Formulations for the HpMP are usually obtained by augmenting a formulation for which feasible solutions correspond to sets of any number of node-disjoint cycles spanning all nodes of the graph with constraints to render solutions with more and less than p cycles infeasible.

We focus on constraints to render solutions with less than p cycles infeasible. We begin by revisiting extended formulations known from [1] which include edge-depot assignment variables. For one of these formulations, the formulations describing the projection of the set of feasible solutions of its Linear Programming (LP) relaxation onto lower dimensional spaces are presented. More specifically, these projections are defined by systems of inequalities including constraints imposing upper bounds on the number of edges used for any tree of the graph. From the other formulation, new effective valid inequalities are obtained. A branch-and-cut algorithm in which these inequalities are separated is tested, with results demonstrating the effectiveness of the new inequalities in improving LP relaxation bounds.

Keywords: *Combinatorial optimization, Mixed integer programming, Polyhedral theory, Branch-and-cut, Hamiltonian p -median*

References

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TB4: Derivative-Free Optimization III

Chair: Yiwen Chen

Room: Santander (Floor 3)

Organizer: Everton Silva & Yiwen Chen

Riemannian optimization with finite-difference gradient approximations

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Abstract

Derivative-free Riemannian optimization aims to minimize an objective function using only function evaluations, under the constraint that the decision variables lie on a Riemannian manifold. The rapid increase in problem dimensions over the years calls for computationally cheap algorithms, that is, algorithms requiring as few function evaluations and retractions as possible. We propose a novel derivative-free Riemannian optimization method based on finite-difference gradient approximations that relies on an adaptive selection of the finite-difference accuracy and stepsize. When endowed with an intrinsic finite-difference scheme, that measures variations of the objective in tangent directions using retractions, our proposed method requires $O(d\epsilon^{-2})$ function evaluations and retractions to find an ϵ -critical point, where d is the manifold dimension. We then propose a variant of our method when the search space is a Riemannian submanifold of an n -dimensional Euclidean space. This variant relies on an extrinsic finite-difference scheme, approximating the Riemannian gradient directly in the embedding space, assuming that the objective function can be evaluated outside of the manifold. This approach leads to worst-case complexity bounds of $O(d\epsilon^{-2})$ function evaluations and $O(\epsilon^{-2})$ retractions. We also present numerical results showing that the proposed methods achieve superior performance over existing derivative-free methods.

Keywords: *Riemannian optimization, Derivative-free, Extrinsic/intrinsic finite difference*

BUP-TR: A Bayesian Underdetermined Projection Framework for Interpolation-Based Trust-Region Optimization

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Abstract

Interpolation-based trust-region (TR) methods are effective for derivative-free optimization but face two bottlenecks in moderate-to-large dimensions: (i) quadratic interpolation is underdetermined when the interpolation set is far smaller than the number of model coefficients, and (ii) geometry management consumes a large fraction of the evaluation budget.

We propose BUP-TR, a framework that resolves both issues via a coefficient-space maximum-a-posteriori (MAP) projection. The underdetermined interpolation system defines an affine feasibility set; BUP-TR selects a unique quadratic model by projecting under a Gaussian prior used as a regularization device. The closed-form completion requires only a small linear solve, and its stability is governed by a computable *MAP-poisedness* condition that unifies model quality and geometry certification.

Under standard assumptions we derive explicit fully-linear error constants and prove global convergence with $\mathcal{O}(\varepsilon^{-2})$ evaluation complexity. The framework is surrogate-agnostic; we instantiate it with a recursive completion prior that propagates accepted MAP coefficients across iterations at zero auxiliary cost. Numerical experiments show statistically significant accuracy gains over the classical minimum-Frobenius baseline, with the advantage growing in dimension and under noise.

Keywords: *Derivative-free optimization, trust-region methods, underdetermined interpolation, MAP projection, geometry management*

References

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Acknowledgments: This work was partially supported by the National Natural Science Foundation of China.

Subspace Modeling and Random Subspace Trust-region Methods in Derivative-free Optimization

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Abstract

Model-based derivative-free optimization (DFO) faces significant challenges in high dimensions due to the cost of constructing accurate interpolation models. Subspace approaches address this by building and optimizing models in low-dimensional affine subspaces. In this talk, we present a unified view of subspace modelling and random subspace trust-region methods. We establish theoretical relationships between full-space and subspace quadratic models and simplex derivatives, showing their consistency on the underlying subspace, and discuss the construction of Q -fully quadratic models for accurate subspace approximations. We then describe random subspace trust-region methods for unconstrained and convex-constrained problems, with convergence guarantees based on subspace-restricted model accuracy and probabilistic subspace quality.

Keywords: *Derivative-free optimization, Large-scale optimization, Subspace approximation, Randomized subspace methods*

Acknowledgments: This research is partially funded by the Natural Sciences and Engineering Research Council of Canada (cette recherche est partiellement financée par le Conseil de recherches en sciences naturelles et en génie du Canada), Discover Grants #2023-03555 and #2024-04643. The first author is also funded by the Killam Doctoral Scholarship and the UBC Okanagan Distinguished Doctoral Scholar Award.

Pareto Surface Navigation for Direct Aperture Optimisation in Radiotherapy

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Abstract

In Intensity-Modulated Radiotherapy, treatment planning involves solving a complex multi-objective optimisation problem that balances tumour coverage and organ-at-risk sparing. Traditional approaches typically rely on Fluence Map Optimisation followed by leaf sequencing, which may degrade plan quality due to the separation between optimisation and deliverability. Direct Aperture Optimisation (DAO) addresses this limitation by incorporating machine deliverability constraints directly into the optimisation process; however, it remains computationally challenging and highly dependent on the choice of objective weights.

In this work, we adapt Pareto Surface Navigation [1] to the DAO setting by explicitly accounting for the structure of aperture-based representations. The proposed approach generates a set of deliverable treatment plans by first obtaining an initial DAO solution using a metaheuristic method. These plans define anchor points on the Pareto surface, over which convex combinations of aperture intensities are used to approximate intermediate solutions while preserving deliverability. This allows navigation across the trade-off surface without the need to resolve the optimisation problem from each weight possible configuration.

Keywords: *Intensity Modulated Radiation Therapy, Direct Aperture Optimisation, Pareto surface navigation, Multi-Objective Optimisation, Radiotherapy*

References

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NSGA-II for Multi-Objective Beam Angle Optimisation in IMRT

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Abstract

In Intensity-Modulated Radiotherapy (IMRT), beam angle optimisation is a complex multi-objective problem that seeks to determine a set of irradiation directions that achieve an appropriate trade-off between tumour coverage and organ-at-risk sparing. Although beam angle optimisation has been widely studied, most existing approaches remain limited in their ability to explicitly address the problem's multi-objective nature. In particular, the discrete and combinatorial structure of the search space makes the design of effective evolutionary operators a central challenge when adapting multi-objective evolutionary algorithms to this setting.

In this work, we study a multi-objective beam angle optimisation approach based on NSGA-II, adapted to the discrete domain of coplanar beam configurations in IMRT. Candidate solutions are encoded as sets of five distinct beam angles selected from a discretised search space, and the algorithm is combined with crossover and mutation operators tailored to preserve feasibility in this representation. The study analyses different operator configurations in order to assess their influence on the quality of the approximation to the Pareto front. This work aims to contribute to the development of more effective evolutionary approaches for multi-objective beam angle optimisation in radiotherapy.

Keywords: *Intensity Modulated Radiation Therapy, Beam angle optimisation, Multi-Objective Optimisation, Non-dominated Sorting Genetic Algorithm II*

Acknowledgments: Supported by ANID Fondecyt Regular 1241710 and ANID Doctorado Nacional 21232361.

Robust Multi-objective Optimisation of Positively Homogeneous Functions with Application to Radiotherapy

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Abstract

Robust multi-objective optimisation aims to account for uncertainty in decision-making, but strict robustness requirements often lead to feasibility issues. In this work, we consider multi-objective problems involving positively homogeneous functions, a class that appears naturally in applications such as radiotherapy treatment planning. Building upon the structural properties introduced in Cabrera-Guerrero et al. [1], we exploit the fact that efficient solutions generate rays in the decision space. We propose a simple feasibility restoration mechanism based on scaling nominal efficient solutions along these rays. For each scenario, a minimal scaling factor is computed, and the robust solution is obtained by selecting the maximum of these values. This reduces the robust feasibility problem to a one-dimensional formulation. The approach is illustrated on the beam angle optimisation problem in IMRT using a Pareto local search framework. Results show that nominally efficient solutions may become infeasible under uncertainty, while the proposed method restores feasibility with a controlled loss in optimality and improves worst-case performance.

Keywords: *Robust optimisation, Multi-objective optimisation, Positive homogeneity, Radiotherapy, Beam angle optimisation*

References

- [1] Cabrera-Guerrero G., et al. (2014). Multi-objective optimisation of positively homogeneous functions.

Acknowledgments: Dr Guillermo Cabrera-Guerrero thanks FONDECYT project 1241710 for partially funding this work.

Hexaly Semi-Plenary**Speaker:** Thierry Benoist**Title:** Beyond traditional MIP**Chair:** Cândida Mourão**Room:** Amphit. 1 (Floor 4)**Organizer:** Hexaly**Abstract**

Hexaly is a new type of global optimization solver. Its modeling formalism is hybrid, unifying concepts from constraint programming, mixed-integer programming, nonlinear programming, and black-box optimization. As a result, it is nonlinear, set-oriented, and supports user-coded functions, enabling seamless integration of simulation with optimization or machine learning with optimization. A central design objective is modeling expressiveness and openness: enabling users to formulate problems closer to their natural combinatorial structure, while allowing diverse algorithmic components to interact in a complementary manner. In this talk, we will present the guiding principles behind this approach, with a particular focus on discrete optimization problems where Hexaly currently demonstrates its strongest performance, such as large-scale routing, scheduling, and packing. We will also explain how Hexaly combines exact and heuristic optimization methods: spatial branch-and-bound, simplex methods, interior-point methods, constraint propagation, automatic branch-cut-price, surrogate modeling, among others.

TC1: Applications II

Chair: Eliana Costa e Silva

Room: Amphit. 2 (Floor 4)

Organizer: Isabel Gomes

Minimizing costs in signal provision by communication antennas along a railway line

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Abstract

In this talk we address a wireless network design problem arising in railway communication systems. This problem originates from a real-world problem proposed by SOLVIT, an engineering company in the Information and Communication Technologies sector, during the 161st European Study Group with Industry. The company sought decision-support tools to design a wireless communication network for railway lines in Portugal. More specifically, given a finite set of candidate locations along a railway line and different types of communication antennas, the objective is to determine which locations and antenna types to select in order to ensure a certain level of signal coverage while minimizing installation costs.

We formulate the problem as a 0/1 linear optimization model, prove that the problem is NP-hard, and report computational experiments using real and simulated data. The computational tests show that the model is capable of solving instances corresponding to railway lines longer than any existing real railway lines. The results presented in this talk were recently published in [1].

Keywords: *Wireless network design; Base station deployment; Railway communications; Optimization; Integer linear programming.*

References

- [1] Araújo, A., Cerdeira, J.O., Lopes, N. et al. Minimizing costs in signal provision by communication antennas along a railway line. 4OR-Q J Oper Res 23, 435–454 (2025).

Acknowledgments: N. Lopes is funded through FCT project UID/PRR/04621/2025 (<https://doi.org/10.54499/UID/PRR/04621/2025>) of CEMAT at Universidade de Lisboa.

Route Optimisation for Sales Teams in the Agricultural Equipment Sector in Brazil

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Abstract

This work addresses a logistical challenge in the context of Nors' Agro segment in São Paulo, Brazil, which specialises in the sale of agricultural equipment, including agricultural tractors, grain harvesters and sugarcane harvesters. Given the geographical dispersion of clients, ranging from highly urbanised metropolitan areas to remote rural regions, planning efficient sales routes presents a significant challenge. The problem consists in defining daily routes for the sales teams, considering different visit frequencies according to the segmentation of the clients. In order to provide the company with a customised solution, two approaches were investigated, based on a real dataset of 300 clients. The initial approach involves the formulation of a mathematical model, incorporating linear programming for each salesperson with a fortnightly approach. The second strategy that was explored consisted in the application of a genetic algorithm, a technique proven effective in the context of salespeople who have an excess of clients. These solutions will assist the company in achieving greater efficiency, with the potential for future implementation in other Nors Group companies.

Keywords: *Vehicle Routing, Industrial Mathematics, Linear Programming, Mathematical Modeling, Genetic Algorithm*

References

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Who (What) is Replenishing the Supermarket Shelves?

Optimization of Hybrid Human–Robot Operations

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Abstract

In-store replenishment is a key driver of retail operational performance and sales. Despite this, it remains highly dependent on human labor, making it increasingly vulnerable to workforce shortages across Europe. These challenges motivate the development of automation-aware optimization models that integrate artificial intelligence and robotics into retail decision-making. Anthropomorphic robotic systems offer the potential to support in-store replenishment operations. However, while warehouses are typically controlled environments, retail stores are inherently designed for human use. As a result, deploying robotic systems in-store introduces additional operational and strategic challenges that must be explicitly addressed. This talk presents results on robotic motion planning in retail environments, together with preliminary findings on planogram optimization. It integrates robotic feasibility analysis with mathematical optimization models to support human–robot–aware planogram design. Preliminary results from a nonlinear shelf allocation model, evaluated using real retail data, show significant variability in task complexity across shelf levels in motion planning experiments based on the HUMP planner. Overall, the work contributes to an optimization framework for hybrid human–robot in-store replenishment systems.

Keywords: *Planogram Optimization, Human-Robot Collaboration, Multi-level Optimization*

Acknowledgments: E. Costa e Silva has been supported by national funds through FCT – Fundação para a Ciência e Tecnologia through project UIDB/ 04728/2025 (<https://doi.org/10.54499/UID/04728/2025>). This research was funded by the Recovery and Resilience Plan (PRR) through the Agency for Competitiveness and Innovation – IAPMEI, I.P, within the scope of the “Agenda PT Smart Retail”, PRR/18_SMARTRETAIL, and supported by FCT – Fundação para a Ciência e Tecnologia within the R&D Unit Project Scope UID/00319/2025 - Centro ALGORITMI (ALGORITMI/UM) <https://doi.org/10.54499/UID/00319/2025>.

TC2: Derivative-Free Optimization IV

Chair: Yiwen Chen

Room: Delta (Floor 3)

Organizer: Everton Silva & Yiwen Chen

Surrogate-based categorical neighborhoods for mixed-variable blackbox optimization

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Abstract

In simulation-based engineering, design choices are often obtained following the optimization of complex black-box models. These models frequently involve mixed-variable domains with quantitative and categorical variables. Unlike quantitative variables, categorical variables lack an inherent structure, which makes them difficult to handle, especially in the presence of constraints.

This work proposes a systematic approach to structure and model categorical variables in constrained mixed-variable blackbox optimization. Surrogate models of the objective and constraint functions are used to induce problem-specific categorical distances. From these distances, surrogate-based neighborhoods are constructed using notions of dominance from bi-objective optimization, jointly accounting for information from both the objective and the constraint functions.

These neighborhoods are employed within CatMADS, an extension of the MADS algorithm for categorical variables. The surrogate models are Gaussian processes, and the resulting method is called CatMADS_{GP}. The method is benchmarked on the Cat-Suite collection of 60 mixed-variable optimization problems and compared against state-of-the-art solvers. Data profiles indicate that CatMADS_{GP} achieves superior performance for both unconstrained and constrained problems.

Keywords: *Blackbox optimization, derivative-free optimization, mixed-variable optimization, constrained optimization, categorical variables*

The quest for descent directions

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Abstract

In this talk, we discuss how to find descent directions of a real-valued function. When a function is differentiable and the gradient is non-zero, then a positive spanning set contains a descent direction. When the function is not differentiable at a point or the gradient is equal to the zero vector, then it is not necessarily true that a positive spanning set contains a descent direction. We will explore what can be said in this case and if it is possible to develop results guaranteeing that a finite set of directions contains a descent direction even when the function is not differentiable. The results may be useful in greybox optimization where the objective function is not known explicitly but some information about the objective function is known.

Keywords: *Greybox optimization, descent directions, positive spanning sets, cosine measure*

NoMoDS: A non-monotone directional direct-search algorithm with worst-case complexity analysis

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Abstract

Directional direct-search methods are an important class of derivative-free, non-model-based methods for minimizing functions when only function values are available. For a subclass based on a sufficient decrease condition, a worst-case complexity (WCC) bound of $\mathcal{O}(n\epsilon^{-2})$ was established in the literature. On the other hand, various non-monotone derivative-free methods, including direct-search variants, have been developed, demonstrating promising numerical performance especially in noisy scenarios and narrow-valley landscapes. In this talk, motivated by the lack of WCC analysis for non-monotone derivative-free methods applied to *general* nonconvex smooth unconstrained optimization problems, we present a Non-Monotone Directional Direct-Search (NoMoDS) algorithm. We provide its global convergence analysis and derive its WCC bound, showing that the algorithm requires at most $\mathcal{O}(n\epsilon^{-2})$ iterations to reduce the gradient norm below ϵ , matching the monotone bound without any complexity penalty.

Keywords: *Directional direct-search, Non-monotone, Worst-case complexity, Derivative-free optimization, Nonconvex*

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Acknowledgments: This work is funded by national funds through the FCT – Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UID/00297/2025 (<https://doi.org/10.54499/UID/00297/2025>) and UID/PRR/00297/2025 (<https://doi.org/10.54499/UID/PRR/00297/2025>) (Center for Mathematics and Applications – NOVA Math).

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Title: Beyond a Single Optimum: Trade-offs and Insights in Multiobjective Optimization

Chair: Ana Luísa Custódio

Room: CGD (Floor 2)

Abstract

This talk gives an introduction to multiobjective optimization. We present fundamental concepts and basic solution approaches, and highlight the additional challenges that arise when moving from one objective to several objectives. In particular, we discuss why the numerical solution of multiobjective problems requires new ideas and present several ingredients that are useful for designing effective solution approaches.

WA1: Production Inventory Routing**Chair:** Rafael Martinelli**Room:** Amphit. 1 (Floor 4)

Balancing inventory and routing in stochastic dynamic inventory-routing: An application to pharmaceutical distribution

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Abstract

This paper studies a stochastic dynamic inventory-routing problem motivated by a pharmaceutical distributor in Portugal that replenishes a set of pharmacies from a central warehouse over a multi-period horizon. In this service-critical setting, routing acts primarily as a resource constraint, while the dominant trade-off lies between inventory holding and stockout costs. To address the complexities of uncertain demand and multi-period routing, we propose an adaptive and anticipatory policy within approximate dynamic programming. It is formulated as a cost function approximation that minimizes inventory deviations from optimized target levels at each pharmacy in a rolling horizon fashion. It also promotes inventory synchronization across neighboring pharmacies by accounting for future routing constraints through a penalty term on inventory imbalances, inspired by the concept of Lagrange multipliers. Both policy parameters, namely the inventory target levels and the Lagrange multiplier, are jointly learned via Bayesian optimization. We evaluate our policy against five benchmarks across six scenarios defined by three levels of demand uncertainty and two vehicle capacity settings. Results show that anticipating future demand via proposed inventory target levels, balancing inventories across neighboring pharmacies, and learning adaptive policy parameters are already effective individually. However, their integration yields superior performance.

Keywords: *Pharmaceutical distribution, Stochastic dynamic inventory-routing, Sequential decision-making, Cost function approximation*

Acknowledgments: This work was supported by FCT - Fundao para a Cincia e Tecnologia [reference: 2022.13466.BD and DOI identifier <https://doi.org/10.54499/2022.13466.BD>]

Robust Optimization algorithms for Multi-Supplier Purchase Planning under Lead Time Uncertainty

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Abstract

The Multiple Suppliers Purchase Planning problem is to select suppliers and schedule orders for getting supplies and minimizing the total purchasing and inventory costs. We consider the problem with a single commodity and lead time uncertainty [1]. We formulate the problem as a two-stage robust optimization model. We study facet-defining inequalities for an uncertainty set similar to the one proposed in [2] and derive the complete polyhedral structure for each single supplier subset. An exact branch-and-cut algorithm based on a Benders-like reformulation and a heuristic framework based on a decomposition of the time horizon are proposed. The computational results show that the exact algorithm can efficiently solve small and medium-size instances, while the heuristic provides good quality solutions for larger instances.

Keywords: *Robust Optimization, Integer Programming, Purchasing Supplier Selection, Lead time uncertainty*

References

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A multi-stage risk-averse stochastic programming algorithm for the perishable production-inventory-routing problem under demand uncertainty

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Abstract

Perishable supply chains face tight shelf-life constraints and volatile demand, requiring coordinated production, inventory, and distribution decisions to reduce spoilage, stockouts, and unnecessary logistics costs. When optimized separately, these decisions create mismatched production quantities, unstable routing plans, and inconsistent replenishment cycles. This paper studies the perishable production-inventory-routing problem using a multi-stage risk-averse static-dynamic stochastic programming framework. In the pre-planning stage, visiting and routing decisions are fixed to stabilize capacity deployment, while chance constraints ensure service reliability. In the multi-period stage, production, inventory, and replenishment decisions adapt to realized demand, while conditional value-at-risk constraints control severe shortages. To solve this NP-hard problem, we develop a reinforcement learning-enhanced progressive hedging (RL-PH) algorithm based on scenario decomposition. Numerical experiments on benchmark instances show that the upper-confidence-bound RL-PH outperforms sample average approximation, random-PH, iterated local search-PH, Q-learning-PH, and DYNA-Q-PH on most instances. Out-of-sample tests confirm the effectiveness of the risk strategies, and sensitivity analysis reveals non-linear effects of shelf-life and storage capacity on cost.

Keywords: *Perishable production-inventory-routing problem, multi-stage stochastic programming, risk-averse optimization, progressive hedging algorithm, reinforcement learning*

A Hybrid Genetic Search for the Inventory Routing Problem

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Abstract

The Inventory Routing Problem (IRP) integrates vehicle routing, delivery scheduling, and inventory decisions over a finite planning horizon. It arises in vendor-managed inventory systems and requires deciding when to serve customers, how much to deliver, and how to route vehicles while minimizing transportation and inventory costs. Despite extensive research, many benchmark instances remain challenging, as highlighted by the 12th DIMACS Implementation Challenge [1].

We propose a Hybrid Genetic Search framework tailored to the IRP [2], combining the exploration capabilities of Hybrid Genetic Search with an exact inventory decoder based on a minimum-cost flow formulation. The method explores routing and scheduling decisions while efficiently computing optimal inventory levels.

We introduce a unified solution representation and a novel crossover operator that builds offspring from period-based giant tours. Inventory levels are computed through a flow-based decoder, followed by route reconstruction. The method also includes improved solution construction and a hybrid local search combining fast routing optimization with full inventory updates, along with acceleration strategies to reduce computational cost.

Experiments on 1,098 benchmark instances show that the method outperforms 22 approaches, producing 137 new best-known solutions on classical sets and 180 on larger instances, and would rank first under DIMACS challenge rules.

Keywords: *Inventory routing problem, Hybrid genetic search, Metaheuristics, Vendor managed inventory*

References

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Hybrid Surrogate for Maximum Throughput determination in Natural Gas Treatment Units

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Abstract

Natural Gas Treatment Units (NGTU) operate under strict product specifications while facing significant variability in feed composition and throughput, often leading to conservative operating strategies and underutilized capacity [1]. This work proposes a hybrid modeling and optimization framework to determine the maximum feasible processing throughput of an NGTU for a given feed composition, combining a data-driven surrogate model with mass balance consistency. The core concept is the prediction of component split factors using a supervised machine learning model trained on reconciled industrial data. All product flow rates and physicochemical properties are subsequently computed through a property layer, ensuring component mass conservation throughout the nonlinear programming (NLP) search space. The hybrid surrogate is embedded in an NLP formulation that identifies the maximum admissible feed rates subject to regulatory quality specifications for sales gas, liquefied petroleum gas (LPG) and natural gasoline. The framework is validated on independent plant scenarios and is compared against a commercial simulator. The results in 14 feed composition scenarios show that the proposed approach identifies throughput limits on average 12.9% higher than conventional fixed-coefficient models, while preserving physical consistency and feasibility. Computational times are faster than simulations, allowing real-time identification of capacity limits and bottlenecks.

Keywords: *Natural Gas Treatment, Nonlinear Programming, Machine learning*

References

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Acknowledgments: The authors acknowledge Petrobras for providing access to industrial operational data that enabled the development of this research.

Learning Decision-Relevant Scenarios for Two-Stage Stochastic Optimization Using Deep Generative Models

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Abstract

In two-stage stochastic optimization problems, decisions must be made based on incomplete realizations of uncertain data in the future. In many practical settings, uncertainty is modeled using a relatively large empirical set of scenarios. However, solving the problem as a whole can become infeasible, especially when decisions are combinatorially complex in both the first and second stages. We introduce the Deep Generative Scenario Search (DGSS) method, which applies deep generative models to construct a small set of synthetic scenarios that capture meaningful information about the underlying problem. It works in an instance-specific way: a generative model is first trained on empirical scenarios, then improved based on feedback from the optimization objective. The reduced stochastic problem for the current set of synthetic scenarios is solved, and the resulting decisions are validated across the entire set of scenarios. As a result, the generator can evolve through a sequence of iterations according to stochastic gradient optimization. Computational studies performed on several NP-hard two-stage stochastic optimization problems (facility location, vehicle routing, and others) confirm that DGSS produces high-quality first-stage solutions using only a few scenarios. As compared to classical sampling techniques, scenario reduction, and learning-based methods, DGSS exhibits more robust and stable performance.

Keywords: *Combinatorial optimization, Stochastic optimization, Deep Generative Models, Scenario Search*

Unboundedness in Bilevel Optimization

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Abstract

Bilevel optimization has garnered growing interest over the past decade. However, little attention has been paid to detecting and dealing with unboundedness in these problems, with most research assuming a bounded high-point relaxation. We address unboundedness in bilevel and multilevel optimization by studying its computational complexity. We show that deciding whether an optimistic linear bilevel problem is unbounded is strongly NP-complete, even without coupling constraints. Furthermore, we extend the hardness result to the linear multilevel case, by showing that for each extra level added, the decision problem of checking unboundedness moves up a level in the polynomial hierarchy. Deciding unboundedness of a mixed-integer multilevel problem is shown to be one level higher in the polynomial complexity hierarchy than the decision problem for linear multilevel problem with the same number of levels. Finally, we introduce two algorithmic approaches to determine whether a linear bilevel problem is unbounded and, if so, return a certificate of unboundedness. This certificate consists of a direction of unboundedness and corresponding bilevel feasible point. We present a proof of concept of these algorithmic approaches on some relevant examples, and provide a brief computational comparison.

Keywords: *Computational Complexity, Unbounded, Bilevel Optimization, Multilevel Optimization*

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Efficient testing of connectivity of a network

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Abstract

We consider a network that represents a geographical area. After a disaster, some arcs of the network may have become unusable. We consider the problem of efficiently determining whether two specific nodes on this network are connected by usable arcs. To do this, it is assumed that we send out drones that spend time reaching the associated arc and conducting tests to find out whether that particular arc is usable or not. The probability that each arc is usable is also known, as all properties of the arcs in the network are known. The problem is to devise a strategy that determines whether the given nodes are connected with the minimum expected cost. This problem is similar to the sequential testing problem studied in the literature, with a different cost structure. We study different versions of the problem that consider series type networks, considering the range of drones and general type networks, and provide solution algorithms. We demonstrate the effectiveness of the algorithms on randomly generated problem instances.

Keywords: *Postdisaster operations, sequential testing, network connectivity*

Acknowledgments: Supported by TUBITAK 1001 program Project number: 224M198

Multilayer Covering Network Design Problems

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Abstract

Multilayer networks are complex structures comprising multiple layers, each containing a distinct network, which are connected with each other. In this paper, we analyze network design problems aiming to maximize trip coverage using multilayer networks. Similar problems have been addressed in the related literature using standard single-layer networks. In this paper, we use, for the first time, multilayer structures for these problems. This approach allows for a more accurate representation of different situations, like multi-modal transportation systems. The nature of links (directed arcs or undirected edges) plays an important role in the decision problem. Therefore, we study two versions of this problem: one with directed arcs and one with undirected edges. We propose mixed integer linear programming programs for both versions of the problem, and several valid inequalities for these models. Finally, some experiments over randomly generated instances assess the models and valid inequalities proposed..

Keywords: *Multilayer network design, covering problem, origin-destination demands*

Acknowledgments: The second and third authors were partially supported by the grants PID2020-114594GB-C21 (MCIN/ AEI /10.13039/501100011033) and PID2024-156594NB-C21 funded by the Spanish Ministerio de Ciencia, Innovación y Agencia Estatal de Investigación. This research has been supported by IMUS-Maria de Maeztu grant CEX2024-001517-M - Apoyo a Unidades de Excelencia María de Maeztu, funded by MI-CIU/AEI/10.13039/501100011033.

Interdiction models for graph propagation

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Abstract

Given a graph $G = (V, E)$ and a set $S \subset V$ of infected nodes, we consider the problem of determining the set of c nodes that minimizes the network propagation on the subgraph that results from the removal of those c nodes. To measure network propagation, we assume that a node i is infected in the resulting subgraph if there is a path from a infected node in S to node i . Several mixed integer programs are presented to find the critical nodes to prevent propagation on directed and undirected graphs. Two compact models are devised by adapting the best-known models for determining a set of critical nodes in a graph to this problem. A new compact model is devised from a network interdiction formulation and a new path-formulation model is introduced. We show that all the models provide the same linear relaxation bound and compare them on a set of instances from the literature. The results show that the compact model from the interdiction perspective is faster, allowing to solve to optimality sparse directed networks with up to 15,000 nodes.

Keywords: *critical nodes, graph propagation, vaccination problem, mixed-integer programming, network interdiction.*

Acknowledgments: CEMS.UL - Center for Mathematical Studies.

Repair sequence problem of electrical transmission networks under uncertainty

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Abstract

Electrical transmission networks are complex systems that deliver power from generation units to primary substations. When disruptions or failures occur, it is essential to have effective strategies in place. These strategies should limit the number of disconnected substations and lessen the impact on other critical infrastructures that depend on them. Besides fixing known faults, it is also important to monitor elements that might fail, allowing for timely interventions.

In this work, we present a Repair Sequence Optimization (RSO) framework designed for electrical transmission networks under uncertainty. This approach combines a mathematical optimization model with a simulation environment developed by ENEA in Italy, in partnership with the transmission system operator Terna S.p.A. The repair sequencing task is set up as a scheduling problem on parallel, non-identical machines, which enables efficient use of repair crews under uncertain processing and travel times and realistic operational constraints. We tested the proposed framework on a realistic transmission network from an Italian region, demonstrating its ability to allocate emergency crews under uncertainty.

Computational experiments, based on randomly generated disruption scenarios, show the effectiveness of the approach. We also analyzed the simulation results to ensure they match practical restoration processes observed in real systems.

Keywords: *Electrical transmission network, Scheduling Model under uncertainty, unrelated parallel machines.*

Optimizing drone-sharing service through flexible hub relocation and mission pooling

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Abstract

This research contributes to the Community Drones[1] framework, a shared infrastructure model that coordinates regional drone services through automated resource sharing. The project improves utility and reduces operational costs by consolidating tasks across diverse sectors and actors. The study addresses a complex drone scheduling problem involving the placement and movement of mobile drone hubs, which serve as temporary bases for drone deployment. Unlike common static models, this approach evaluates relocating hubs to expand coverage and integrating multiple jobs into combined missions. This allows a single drone flight to perform multiple tasks, improving resource utilization and reducing overall operational time.

We formulate this problem as a flexible job shop scheduling problem (FJSP), considering hubs as reconfigurable resources. To solve this, we first present an integer linear programming (ILP) model to derive exact solutions for small-sized instances. To address the computational complexity of large-scale regional deployments, we develop a heuristic approach based on advanced neighborhood search techniques. Computational experiments based on simulated regional demand scenarios show that the mobile hub model increases mission completion rates while reducing completion times and operational costs compared to static configurations. This research provides a scalable solution for regional authorities seeking to implement efficient, cross-sector drone infrastructure.

Keywords: *drone-sharing, drone scheduling, hub relocation, neighborhood search*

References

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Acknowledgments: This work was supported by the Austrian Federal Ministry of Climate Action (BMK) through the FFG ‘Take Off’ program (Grant No. FO999913929).

Parallel machine scheduling with multiple orders per job and batching

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Abstract

In modern 300-mm wafer fabs, the front-opening unified pod (FOUP) is the standard transfer unit. Large wafer sizes allow customer orders to be fulfilled with partial FOUPs, necessitating the grouping of multiple orders into a single job. This challenge is known as the multiple orders per job (MOJ) scheduling problem. Efficient FOUP utilization while maintaining system performance is critical for production efficiency. This study investigates the MOJ scheduling problem with batching in an identical parallel machine environment to minimize total weighted tardiness (TWT). We propose a multi-stage heuristic framework: (1) order sorting and grouping into jobs, (2) job sequencing, and (3) forming jobs into batches for machine assignment. Unlike existing literature, this research explicitly incorporates batching constraints to better reflect practical fab operations. A mathematical model is provided to evaluate the problem structure and provide optimal solutions for small-scale instances. To verify the approach, computational experiments were conducted. Our heuristics were compared against common industrial dispatching rules, including Earliest Due Date (EDD) and First-Come-First-Served (FCFS). Results demonstrate that by incorporating batching logic and systematic sorting stages, our method significantly outperforms traditional rules in minimizing TWT. This research offers a practical scheduling procedure to enhance efficiency in parallel machine work centers.

Keywords: *Scheduling, parallel machines, multiple orders per job, batch, heuristics*

Acknowledgments: This research was supported by the National Science and Technology Council, Taiwan, under Grant No. NSTC 114-2221-E-035 -073 -MY3.

A comparative study on models for optimizing tardiness on parallel machines with sequence-dependent setup times and an extension by special jobs with external delivery dates

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Abstract

Scheduling on identical parallel machines remains a key challenge in modern production systems, particularly when sequence-dependent setup times and delivery reliability must be considered. This paper conducts a comprehensive analysis of four types of optimization models - sequence-based, position-based, time-indexed, and network-based – that are adapted from the literature and systematically evaluated through extensive computational studies. We analyze optimization model performance across benchmark instances comprising up to 80 jobs and 16 machines under limited runtime conditions, using different commercial and open-source solvers. Three tardiness-related objective functions are examined: total tardiness, total weighted tardiness, and total quadratic tardiness. The comparative analysis identifies the most efficient combination of model type and solver in terms of computational efficiency and scalability. In addition, we propose an extension to the basic scheduling problem that incorporates external jobs related to delivery date schedules, e.g., imposed by third-party logistics providers. This extension distinguishes between internal jobs with common due dates and external jobs with delivery due dates. A weighting factor enables the explicit integration of job-type preferences into the scheduling process. Computational experiments demonstrate the reasonability of the extension and reveal its sensitivity to the share of external jobs and the weighting factor. The results provide valuable insights into the interdependence of production scheduling and delivery execution.

Keywords: *scheduling, identical parallel machines, sequence-dependent setup times, delivery dates, mixed-integer programming*

WA5: Global Optimization

Chair: Ana Rocha

Organizer: Ana Rocha

Room: Santander (Floor 3)

A Variational Model for Equilibrium Problems in Cloud Data Centers

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Abstract

The aim of this talk is to introduce a variational equilibrium model for resource allocation problems arising in cloud data centers [1]. The system is modeled as a network of interacting agents, where each server is evaluated through trust and reputation mechanisms and seeks to maximize a performance measure defined as the difference between benefits and costs.

The reputation of each agent is described as a weighted aggregation of the trust values it receives, where the weights depend on the relative distances among agents. This structure gives rise to a coupled equilibrium problem, in which the outcome associated with each agent depends on the evaluations provided by the others.

The model is formulated within a variational framework and analyzed using tools from the theory of variational inequalities. Conditions ensuring existence and uniqueness of the equilibrium solution are discussed.

Numerical experiments are presented to illustrate the qualitative behavior of the model and to assess the influence of key parameters on the resulting equilibrium configuration.

Keywords: *Trust and reputation systems, Equilibrium conditions, Variational inequalities*

References

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Equilibrium Models for Global Optimization in Pharmacy Networks

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Abstract

This work studies a spatial optimization problem for the design of pharmacy networks in regional healthcare systems, with the aim of improving accessibility and service equity. We formulate a global optimization problem, consistent with decentralized user behavior, based on variational inequality theory. Users are modeled as rational agents selecting pharmacies by minimizing generalized costs, including travel distance and service attractiveness, under spatial constraints. Compared to classical location models, the proposed approach explicitly incorporates interactions among agents and captures system-level effects induced by individual choices. A numerical example is provided to illustrate the model structure and to demonstrate its applicability in supporting strategic planning in pharmaceutical distribution systems.

Keywords: *Pharmaceutical supply chain, Equilibrium problems, Variational inequality.*

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On polytopal spatial branch and bound

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Abstract

Branch and bound (B&B) Global Optimization algorithms aim at enclosing the set of minimizer points and the minimum function value up to guaranteed accuracy. Traditionally, the partition sets in spatial B&B are boxes, simplices or cones. Our study on simplicial B&B has shown that simplices may better cover a polytope feasible set than boxes and capture minimum points at its relative boundary than boxes that have to deal with feasibility issues, see [2]. On the other hand, in a B&B tree, boxes reveal a better volume reduction in bisection oriented refinement, which is mainly an advantage when the dimension of the feasible set is relatively high.

Therefore, we have an interest in the question of using polytopes directly as partition sets. On the one hand, as each partition set is feasible, it has a clear advantage over box shaped partition sets. On the other hand, using a coordinate wise bisection provides a better volume reduction than using simplicial partition sets, see [3]. However, despite the theoretical attractiveness, the practical elaboration of polytopal algorithms is a big challenge, see [2]. In this presentation, I will share the experience we got in getting polytopal B&B to work and the difficulty to compare its behavior with other types of algorithms.

Keywords: *branch and bound, Global optimization, simplex, polytope*

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On Augmented and Penalty Weighted Tchebycheff Scalarizing Functions for Multiobjective Optimization

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Abstract

When solving a multiobjective optimization problem (MOP) no single optimal solution exists. Instead, a set of non-dominated solutions can be obtained, the well-known Pareto optimal front (POF). To solve a MOP, a decomposition-based approach decomposes a MOP into a set of scalar optimization subproblems and optimizes them (simultaneously) in a collaborative manner, where the objective in each subproblem is an aggregation of all the objectives [1], known as a scalarizing function (SF). The choice of SF directly affects convergence behavior and the distribution of solutions along the Pareto front. Since the position of the contour lines of the SF is crucial, augmented and penalty-based weighted Tchebycheff SFs, based on a normalized weight vector, are proposed and tested for different values of a user defined parameter. The influence of this parameter on the opening angle of the contour lines and on the improvement region of a solution is analyzed, in order to promote convergence to the POF and maintain diversity of solutions on the POF [2,3,4].

Keywords: *Multiobjective optimization, Tchebycheff scalarization, Penalty strategy*

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Designing Telemedicine Clinic Networks for Rural Follow-Up Care

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Abstract

Telemedicine clinics can strengthen access to planned follow-up care in rural areas by bringing supervised consultations closer to patients and reducing unnecessary travel to regional hospitals, especially for people with chronic conditions. This study develops an optimization framework for planning telemedicine clinic deployment in rural healthcare systems. The model decides which candidate clinics to open and how much capacity to assign to each site while accounting for the continuing role of the regional hospital. Patient choice between the hospital and telemedicine clinics is captured through a distance-based random utility model, allowing demand to emerge endogenously. Congestion effects are incorporated through queueing-based waiting-time relationships within a mixed-integer conic optimization model. A Danish rural case study shows that the framework can identify both consistently attractive clinic locations and more context-dependent ones. The results highlight trade-offs among accessibility, congestion, utilization, and equity, and provide decision support for integrating telemedicine clinics with hospital-based follow-up care in rural regions.

Keywords: *OR in health services, Telemedicine clinics; Patient choice; Congestion; Healthcare in rural area.*

A Simulation Framework for the Multi-Period Vehicle Routing Problem with Profits in Smart Waste Collection

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Abstract

Urban waste collection is a critical municipal service with profound implications for sanitation and public health. Traditionally, it has been modeled as a static Capacitated Vehicle Routing Problem (CVRP), where vehicles perform exhaustive collection of all bins. The introduction of smart bin monitoring enables dynamic routing, allowing vehicles to visit only bins that require immediate service or are profitable to collect, modeled as a Vehicle Routing Problem with Profits (VRPP). This must be performed over multiple periods, where routes in each period affect subsequent periods, forming a Multi-Period VRPP (MPVRPP). Due to scale and temporal dependencies, adapting CVRP solvers is challenging, and progress is hindered by the lack of standardized multi-period evaluation environments. To address this, we propose a Python simulation framework for MPVRPP and a methodology to adapt CVRP algorithms, including Hybrid Genetic Search (HGS), Adaptive Large Neighborhood Search (ALNS), and Neural Combinatorial Optimization (NCO). Adaptations include profit-aware repair operators and dynamic dispatch heuristics. An extensive benchmark evaluates the solvers, establishing a unified baseline and demonstrating the framework as a testbed for future research in dynamic, multi-period waste collection routing.

Keywords: *Multi-Period Vehicle Routing Problem; Smart Waste Collection; Simulation Framework; Neural Combinatorial Optimization; Benchmarking*

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Reducing the Carbon Footprint of Multi-Compartment Vehicles

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Abstract

This work addresses the Multi-Compartment Vehicle Routing Problem (MCVRP) with a focus on environmental sustainability, relevant for transporting goods that must remain separated because of the distinct temperature requirements. We consider a homogeneous fleet consisting of multi-compartment vehicles, each equipped with both room temperature and frozen compartments. The distribution scenario involves multiple pick-up points and a single logistics center. The primary objective is to minimize total CO₂ equivalent emissions, which arise from two main sources: (1) fuel or electricity consumption during transportation, which depends on travel distance; and (2) energy consumption during vehicle setup time related to the preparation of frozen compartments whose pre-cooling duration varies based on compartment size. To address this, we propose a three-step hybrid solution approach. First, demand points are grouped in an optimal number of clusters, considering spatial proximity. Then, we perform route optimization within each cluster, taking into account vehicle capacity, demand compatibility, and fuel efficiency. In the final step, each cluster is treated as a single node, and inter-cluster routing is optimized to minimize total emissions from inter-cluster travel and setup operations. Linear programming is applied in the latter two steps. This method combines the strengths of exact and heuristic techniques, delivering environmentally sustainable solutions while minimizing the carbon footprint of fleet utilization.

Keywords: *controlled-temperature vehicle routing, clustering, linear programming, CO₂eq emissions*

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Title: Some Thoughts on Duality in Continuous and Discrete Optimisation

Chair: Luís Gouveia

Room: CGD (Floor 2)

Abstract

Duality is a fundamental concept in optimisation. In the first half of this talk, I will give a gentle introduction to duality in continuous and discrete optimisation. Examples will include (i) classic linear programming duality; (ii) Lagrangian duality for nonlinear programming; (iii) duality for conic optimisation; and (iv) Lagrangian, surrogate and subadditive duality for integer programming.

In the second half of the talk, I will review and discuss various ways in which duality can be of practical use. Examples will include (i) sensitivity analysis; (ii) the design of exact or heuristic algorithms for various problems; and (iii) theorem-proving. The last of these three applications may be surprising to some.

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